

$$\text{HW 7 \# 2b)} \int_{\gamma} \frac{1}{z^2 - 2z - 8} dz$$

$\gamma$  circle of radius 3  
centered at 0

$$f(z) = \frac{1}{z^2 - 2z - 8} \quad \text{where is that holomorphic?}$$

everywhere where  $z^2 - 2z - 8 \neq 0$

$$\text{Solve } z^2 - 2z - 8 = 0$$

$$z^2 - 4z + 2z - 8 = 0$$

$$z(z - 4) + 2(z - 4) = 0$$

$$(z + 2)(z - 4) = 0$$

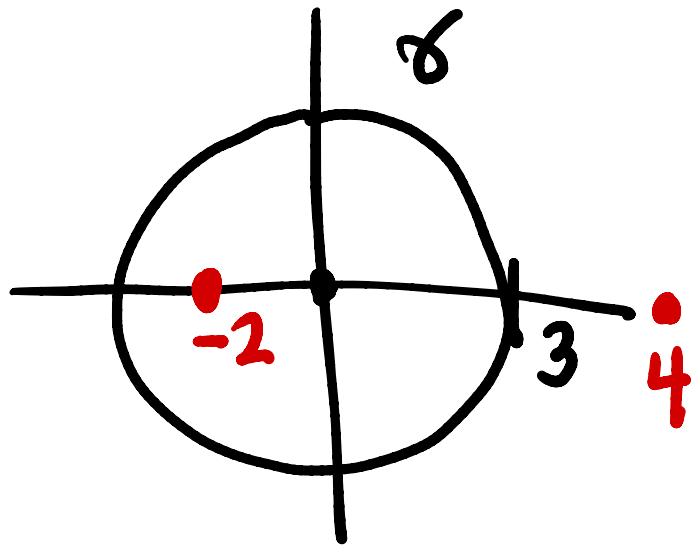
$$S = -2 = -4 + 2$$

$$P = -8 = (-4)(2)$$

$$z = -2$$

$$z = 4$$

$f = \frac{1}{z^2 - 2z - 8}$  is holomorphic in  $U = \mathbb{C} - \{-2, 4\}$



1<sup>st</sup> way:

Do PFD to write

$$\frac{1}{z^2 - 2z - 8} = \frac{A}{z+2} + \frac{B}{z-4}$$

$$\int_{\delta} \frac{1}{z^2 - 2z - 8} dz = \int_{\delta} \frac{A}{z+2} dz + \int_{\delta} \frac{B}{z-4} dz$$

2<sup>nd</sup> way: Notice that

$$\frac{1}{z^2 - 2z - 8} = \frac{\frac{1}{z-4}}{z+2}$$

$$\int_{\gamma} \frac{1}{z^2 - 2z - 8} dz = \int_{\gamma} \frac{\frac{1}{z-4}}{z+2} dz$$

$$= \frac{1}{z-4} \div \frac{z+2}{1}$$
$$= \frac{1}{z-4} \cdot \frac{1}{z+2}$$

CIF  $f(z) = \frac{1}{z-4}$  holomorphic interior of  $\gamma$

$$\text{So } f(-2) = \frac{1}{2\pi i}$$

$$\int_{\gamma} \frac{f(z)}{z - (-2)} dz$$

$$\text{So } \int_{\gamma} \frac{1}{z^2 - 2z - 8} dz = \int_{\gamma} \frac{1}{\frac{z-4}{z+2}} dz$$

$$= 2\pi i \cdot f(-2) \quad f(z) = \frac{1}{z-4}$$

$$= 2\pi i \cdot \frac{1}{-6}$$

$$= \frac{-\pi i}{3}$$

$$\frac{1}{z^2 - 2z - 8} = \frac{A \cdot \overbrace{z-4}^{z-4}}{z+2} + \frac{B \cdot \overbrace{z+2}^{z+2}}{z-4}$$

$$\frac{1}{(z+2)(z-4)} = \frac{A(z-4) + B(z+2)}{(z+2)(z-4)}$$

$$\Rightarrow 1 = Az - 4A + Bz + 2B$$
$$0z + 1 = (A+B)z + 2B - 4A$$

$$\Rightarrow \begin{aligned} A+B &= 0 & A &= -B \\ 2B-4A &= 1 & \checkmark & \\ 2B+4B &= 1 & B &= \frac{1}{6} \end{aligned}$$

$$A = -\frac{1}{6}$$

$$\frac{1}{z^2 - 2z - 8} = \frac{-1/6}{z+2} + \frac{1/6}{z-4}$$



$$\int_{\gamma} \frac{1}{z^2 - 2z - 8} dz = \int_{\gamma} \frac{-1/6}{z+2} dz + \int_{\gamma} \frac{1/6}{z-4} dz$$

holomorphic inside  $\gamma$   
so contract  $\gamma \sim 0$

$$\int_{\gamma} \frac{f(z)}{z-z_0} dz$$

$$f(z) = -1/6$$
$$z_0 = -2$$

$$= 2\pi i f(-2) = 2\pi i \left(-\frac{1}{6}\right) = -\frac{\pi i}{3}$$