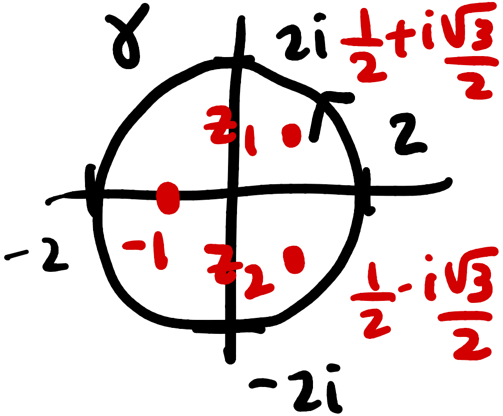
HW7 1c) Show that $\int_\gamma \frac{1}{z^3+1}\,dz = 0$



Where is $f(z) = \frac{1}{z^3+1}$ holomorphic?

Where denominator $\neq 0$

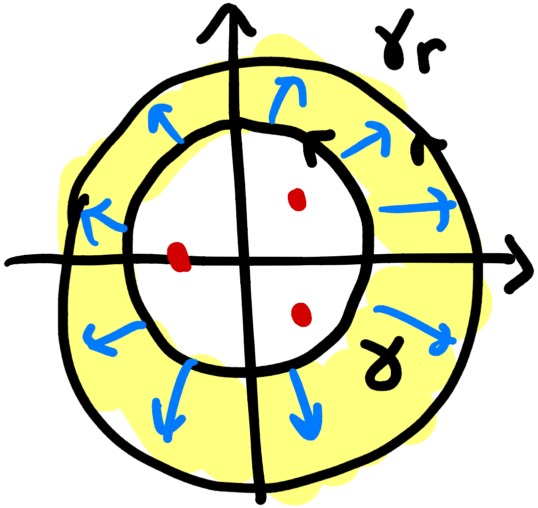
$$z^3+1=0$$
$$z^3=-1$$

f is holomorphic on $\mathcal{U} = \mathbb{C} - \{-1, z_1, z_2\}$

i. Let γ_r circle of radius r centered at 0, $r \geq 2$

$\gamma = \gamma_2$

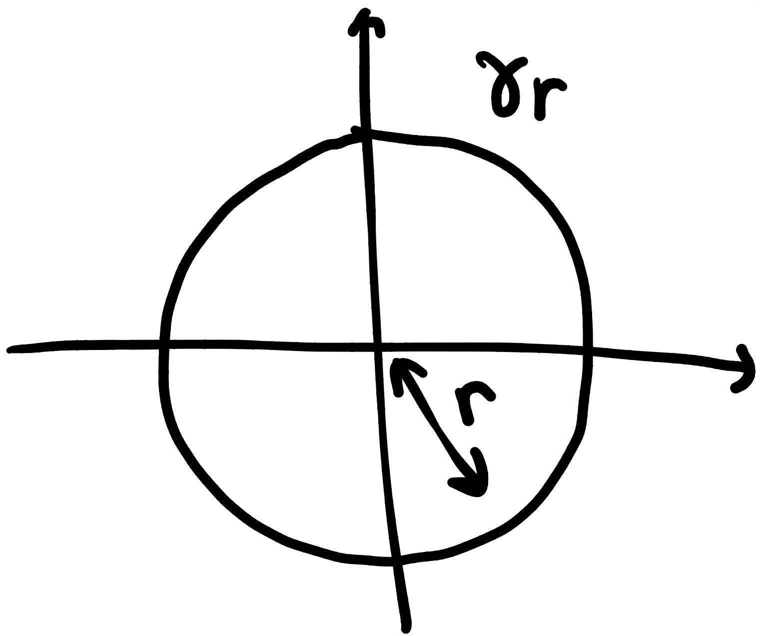
$\gamma_r \sim_u \gamma$



By Cauchy's Theorem

$$\int_{\gamma_r} \frac{1}{z^3+1}\,dz = \int_{\gamma} \frac{1}{z^3+1}\,dz$$

ii. $\left| \int_{\gamma_r} \frac{1}{z^3+1}\, dz \right| \leq$ length of γ_r · max value of $|f|$ on γ_r



γ_r is a circle of radius r
so its length is $2\pi r$

$$\left| \frac{1}{z^3+1} \right| = \frac{1}{|z^3+1|}$$

$$\left|\frac{1}{z^3+1}\right| = \frac{1}{|z^3+1|} \leq \frac{1}{|z|^3-1} = \frac{1}{r^3-1}$$

Need: $|z^3+1| \geq$??

on γ_r $|z|=r$

Reverse Δ inequality

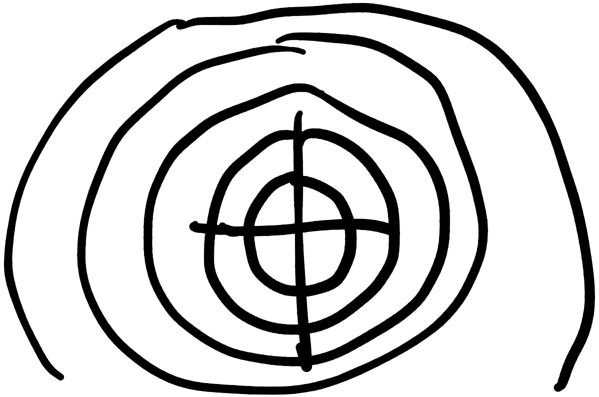
$$|z^3+1| \geq |z|^3 - 1$$

$$\left|\int_{\gamma_r} \frac{1}{z^3+1}\,dz\right| \leq 2\pi r \cdot \frac{1}{r^3-1}$$

iii. $\lim_{r\to\infty} \frac{2\pi r}{r^3-1} = \lim_{r\to\infty} \frac{2\pi}{r^2-\cancelto{0}{\frac{1}{r}}} = 0$

iv. $\left| \int_{\gamma} \frac{1}{z^3+1}\,dz \right| = \left| \int_{\gamma_r} \frac{1}{z^3+1}\,dz \right| \leq \frac{2\pi r}{r^3-1}$

original path



$$\left| \int_\gamma \frac{1}{z^3+1}\,dz \right| \le \frac{2\pi r}{r^3+1} \quad \text{for every single } r \ge 2$$

The only nonnegative number that is less than any positive number is 0

So $$\int_\gamma \frac{1}{z^3+1}\,dz = 0$$