Math 295 - Fall 2020
Homework 5
Due at 11:59pm on Friday October 9

Please turn in this assignment on Gradescope.

Problem 1 : (Objective C5)

a) Notice how Bowman defines the principal square root $\sqrt{z}$. How would Bowman define the principal nth root $z^{1/n}$?

b) How does BMPS define $z^{1/n}$?

c) Are these two different or the same?

d) Find a subset U of $\mathbb{C}$ whose image under the function $f(z) = z^n$ for n a positive integer covers all of $\mathbb{C}$ except 0 and the negative real axis.

Problem 2 : (Objectives C6, C7, C8) Convert the following expressions to the form $x + iy$. (Reason carefully, and use the BMPS definition of complex exponents.)

a) $e^{i\pi}$

b) e^{π}

c) $\exp(\text{Log}(3 + 4i))$

d) $\text{Log}(\exp(3 + 4i))$

e) i^i

Problem 3 : (Objective C7)

a) Compute $\text{Log}((1 - i\sqrt{3})^n)$ for $n = 1, 2, 3, 4$.

b) What do you notice? Does this agree with the properties of the real logarithm function?

Problem 4 : (Objectives C6, C7, C8) Find all solutions to the following equations:

a) $\text{Log}(z) = \frac{\pi i}{2}$

b) $\text{Log}(z) = \frac{3\pi i}{2}$

c) $\exp(z) = \pi i$

d) $z^{1/2} = 1 + i$

Problem 5 : (Objective C6) Prove that $\exp(b \log a)$ is single valued if and only if b is an integer. Note that since a^b is defined to be this expression, it means that the expression z^n is well defined in a polynomial, no matter which branch of the logarithm we use to compute it.