Math 295 - Fall 2020
Homework 10
Due at 11:59pm on Friday November 20

Please turn in this assignment on Gradescope.

Problem 1 : (Objective F1) Find the poles or removable singularities of the following functions and determine their orders.

a) $(z^2+1)^{-3}(z-1)^{-4}$

b) $z^{-5}\sin(z)$

c) $\dfrac{1}{1-\exp(z)}$

d) $\dfrac{z}{1-\exp(z)}$

Problem 2 : (Objective F2) Evaluate the following integrals.

a) $\displaystyle\int_\gamma z^2 \exp\left(\frac{1}{z}\right) dz$, where γ parametrizes the circle of radius 3 centered at 0

b) $\displaystyle\int_\gamma z^3 \cos\left(\frac{3}{z}\right) dz$, where γ parametrizes the circle of radius 3 centered at 0

Problem 3 : (Objective F3) In this problem we will compute the residue at 0 of the following function

$$f(z) = \frac{z^2+4z+5}{z^2+z}$$

in three different ways.

a) Compute the residue of f using the power series for $\dfrac{1}{1-z}$ and the fact that

$$\frac{z^2+4z+5}{z^2+z} = (5+4z+z^2)\cdot\frac{1}{1-(-z)}\cdot\frac{1}{z}.$$

b) What is the order of the pole of f at $z=0$? Compute the residue of f using Proposition 9.11 of BMPS.

c) Compute the residue of f using Proposition 9.14 of BMPS.

d) Which method did you find to be easiest? Hardest?

Problem 4 : (Objective F3) Evaluate the following integrals using the technique of your choice.

a) $\displaystyle\int_\gamma \frac{dz}{(z+4)(z^2+1)}$, where γ is the circle of radius 3 centered at 0

b) $\displaystyle\int_\gamma \frac{\exp(z)}{(z+2)^2\sin(z)}\,dz$, where γ is the circle of radius 3 centered at 0

c) $\displaystyle\int_\gamma \frac{\exp(4z)-1}{\sin^2(z)}\,dz$, where γ is the unit circle centered at 0

Problem 5 : (Objective Grad2) Show that if f has an essential singularity at z_0 then $\dfrac{1}{f}$ also has an essential singularity at z_0.