

Math 395 - Fall 2019
Midterm Exam

Please solve **BOTH** problems below:

1. Let G be a group of order 105 and assume that G contains a subgroup N of order 15.
 - (a) Explain why N is cyclic.
 - (b) Show that if G does not have a normal 7-Sylow subgroup, then N is normal in G .
 - (c) Assume N is normal in G . By considering the action of G on N by conjugation, show that N is contained in the center of G , and then show that G is cyclic.

2. Let p be a prime and let P be a p -group acting on a nonempty finite set A with $(\#A, p) = 1$.
 - (a) Prove that there is some $a \in A$ that is fixed by every element of P .
 - (b) Suppose P is a p -subgroup of a finite group G and H is a normal subgroup of G with $(\#H, p) = 1$. Deduce from (a) that for every prime q dividing $\#H$ there is a Sylow q -subgroup of H that is normalized by P .