

Shorthand notation:

①

We write $A\vec{x} = \vec{b}$ to mean the system:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m = b_2$$

$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nm}x_m = b_n$$

The matrix A is the matrix of coefficients:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \in \mathcal{M}_{n \times m}$$

The vector \vec{x} is the vector of variables:

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \in \mathbb{R}^m$$

The vector \vec{b} is the vector of constants:

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \in \mathbb{R}^n$$

Example: The system

$$x_1 + 4x_2 + 3x_3 = 1$$

$$6x_1 + 7x_2 + 8x_3 = 2$$

is $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 6 & 7 & 8 \end{pmatrix}$$

\in

$\mathcal{M}_{2 \times 3}$

$$\vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

\in

\mathbb{R}^2

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

\in

\mathbb{R}^3