Summary: Kinds of Solution Sets

		number of solutions of the homogeneous system	
		one	*infinitely many*
particular solution exists?	*yes*	unique solution	infinitely many solutions
	no	no solutions	no solutions

An important special case is when there are the same number of equations as unknowns.

3.11 *Definition* A square matrix is *nonsingular* if it is the matrix of coefficients of a homogeneous system with a unique solution. It is *singular* otherwise, that is, if it is the matrix of coefficients of a homogeneous system with infinitely many solutions.

Planes

The plane in $\mathbb{R}^3$ through the points $(1, 0, 5)$, $(2, 1, -3)$, and $(-2, 4, 0.5)$ consists of (endpoints of) the vectors in this set.

$$\{\begin{pmatrix}1\\0\\5\end{pmatrix} + t\begin{pmatrix}1\\1\\-8\end{pmatrix} + s\begin{pmatrix}-3\\4\\-4.5\end{pmatrix} \mid t, s \in \mathbb{R}\}$$

The column vectors associated with the parameters

$$\begin{pmatrix}1\\1\\-8\end{pmatrix} = \begin{pmatrix}2\\1\\-3\end{pmatrix} - \begin{pmatrix}1\\0\\5\end{pmatrix} \qquad \begin{pmatrix}-3\\4\\-4.5\end{pmatrix} = \begin{pmatrix}-2\\4\\0.5\end{pmatrix} - \begin{pmatrix}1\\0\\5\end{pmatrix}$$

each have their whole body in the plane.

A set of the form $\{\vec{p} + t_1\vec{v}_1 + t_2\vec{v}_2 + \cdots + t_k\vec{v}_k \mid t_1, \ldots, t_k \in \mathbb{R}\}$ where $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$ and $k \leqslant n$ is a k-*dimensional linear surface* (or k-*flat*).

Length and angle measures

Length

2.1 *Definition* The *length* of a vector $\vec{v} \in \mathbb{R}^n$ is the square root of the sum of the squares of its components.

$$|\vec{v}\,| = \sqrt{v_1^2 + \cdots + v_n^2}$$

Example The length of

$$\begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$$

is $\sqrt{1+4+9} = \sqrt{14}$.

For any nonzero vector $\vec{v}$, the length one vector with the same direction is $\vec{v}/|\vec{v}|$. We say that this *normalizes* $\vec{v}$ to unit length.