Name:

Problem 1: *Solve the following system of linear equations:*

$$\begin{aligned} x - z &= 0 \\ 3x + y &= 1 \\ -x + y + z &= 4 \end{aligned}$$

Solution: We first put the system in echelon form:

$$\begin{aligned} x - z &= 0 \\ 3x + y &= 1 \\ -x + y + z &= 4 \end{aligned} \quad \overset{\rho_2 - 3\rho_1}{\underset{\rho_3 + \rho_1}{\longrightarrow}} \quad \begin{aligned} x - z &= 0 \\ y + 3z &= 1 \\ y &= 4 \end{aligned} \quad \overset{\rho_3 - \rho_2}{\longrightarrow} \quad \begin{aligned} x - z &= 0 \\ y + 3z &= 1 \\ -3z &= 3 \end{aligned}$$

Now that the system is in echelon form we just have to "walk up":

- From the bottom row we get $z = -1$.

- Then plugging this into the second row we get $y - 3 = 1$ or $y = 4$.

- Finally plugging this into the first row we get $x - (-1) = 0$ or $x = -1$.

There is a unique solution and it is $(-1, 4, -1)$.