MATH 124 QUIZ 23 NOVEMBER 30 2016

Name:

Problem 1: *Consider the linear map*

$$f\colon \mathbb{R}^2 \to \mathbb{R}^3$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ x-y \\ 3y \end{pmatrix}.$$

a) Give the matrix representation of this map. We will call this matrix A.

$$\begin{pmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 3 \end{pmatrix}$$

b) Put A in echelon form.

$$\begin{pmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 3 \end{pmatrix} \overset{\rho_1 \to \rho_3 \to \rho_2}{\sim} \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 0 & 0 \end{pmatrix}$$

c) In your echelon form above, how many variables are leading? How many variables are free?

Both x and y are leading variables. There are no free variables.

d) What is the rank of f? You do not need to justify your answer.

There are two leading variables in the echelon form of the matrix associated to f, and therefore the rank of f is 2.

e) What is the nullity of f? You do not need to justify your answer.

There are no free variables in the echelon form of the matrix associated to f, and therefore the nullity of f is 0.