Name:

Problem 1: *Consider the set*

$$S = \left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\-3\\7\end{pmatrix} \right\}.$$

Give a subset of S that is linearly independent but that spans the same space.
*You **must** show your work to receive any credit.*

Solution: We first solve the equation

$$a_1 \begin{pmatrix}1\\2\\3\end{pmatrix} + a_2 \begin{pmatrix}4\\5\\6\end{pmatrix} + a_3 \begin{pmatrix}0\\1\\0\end{pmatrix} + a_4 \begin{pmatrix}1\\-3\\7\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}.$$

We do this by solving a homogenous system. Here we only give the row operations, and we omit the column of zeroes:

$$\begin{pmatrix}1 & 4 & 0 & 1\\2 & 5 & 1 & -3\\3 & 6 & 0 & 7\end{pmatrix} \underset{\rho_3-3\rho_1}{\overset{\rho_2-2\rho_1}{\sim}} \begin{pmatrix}1 & 4 & 0 & 1\\0 & -3 & 1 & -5\\0 & -6 & 0 & 4\end{pmatrix} \overset{\rho_3-2\rho_2}{\sim} \begin{pmatrix}1 & 4 & 0 & 1\\0 & -3 & 1 & -5\\0 & 0 & -2 & 14\end{pmatrix}$$

From this we see that a_1, a_2 and a_3 are leading and a_4 is free. We conclude that

$$\operatorname{Span}\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\-3\\7\end{pmatrix} \right\} = \operatorname{Span}\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \right\}$$

and

$$\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \right\}$$

is linearly independent. In other words, the fourth vector was superfluous.