Name:

Problem 1: *Is the set*

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \right\}$$

linearly independent or linearly dependent?

Solution: We investigate the solutions to the equation

$$a_1 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + a_2 \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Writing it out, this means that we are looking for a_1 and a_2 that are solutions of this system:

$$\begin{aligned} a_2 &= 0 \\ 0 &= 0 \\ -a_1 + 4a_2 &= 0. \end{aligned}$$

From the first equation we see immediately that $a_2 = 0$, and plugging into the last equation we see that $a_1 = 0$ as well. This is the only solution.

Since the trivial solution $a_1 = a_2 = 0$ is the only solution to the equation above, the two vectors are linearly independent.