1. Let

$$\begin{aligned} f\colon \mathbb{R}^2 &\to \mathbb{R}^3 \\ \begin{pmatrix} x \\ y \end{pmatrix} &\mapsto \begin{pmatrix} x+y \\ 0 \\ 2x-y \end{pmatrix}. \end{aligned}$$

and

$$\begin{aligned} g\colon \mathbb{R}^3 &\to \mathbb{R}^2 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} &\mapsto \begin{pmatrix} x+y \\ x+z \end{pmatrix}. \end{aligned}$$

   (a) Give the matrix representation of f.

   (b) Give the matrix representation of g.

   (c) Give the matrix representation of $g \circ f$. Call this matrix A.

   (d) Let
   $$\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

      i. Compute $f(\vec{v})$. Call this vector $\vec{w}$.

      ii. Compute $g(\vec{w})$.

      iii. Compute the matrix-vector product $A\vec{v}$.

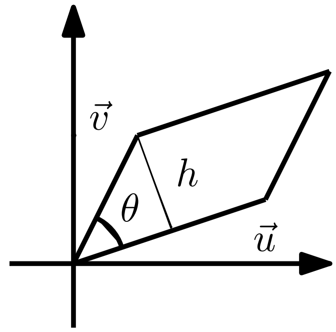
2. Let
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

   (a) Compute A^{-1}.

   (b) Use A^{-1} to compute the solution set of the system
   $$\begin{aligned} x \phantom{{}+2y} + z &= 2 \\ x + 2y - z &= 0 \\ -y + 2z &= -4 \end{aligned}$$

   (Hint: This is the system $A\vec{x} = \vec{b}$ for some $\vec{b}$.)

3. Consider the parallelogram whose four corners are at the points $A = (0,0)$, $B = (3,1)$, $C = (1,2)$ and $D = (4,3)$. We will call $\vec{u}$ the vector going from A to B (so $\vec{u} = \left(\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}\right)$) and we will call $\vec{v}$ the vector going from A to C (so $\vec{v} = \left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)$). Finally, we denote the angle between the vectors $\vec{u}$ and $\vec{v}$ by θ and the height of the parallelogram by h.



(a) What is $\cos\theta$?

(b) What is $\sin\theta$? (Hint: You don't need to know θ to get $\sin\theta$.)

(c) What is the length of h? (Hint: You will need $\sin\theta$ and $|\vec{v}|$.)

(d) What is the area of the parallelogram? (Hint: The area of a parallelogram is height times base, and here the base is $\vec{u}$.)

(e) Compute the determinant

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}.$$

4. In this problem, let

$$A = \begin{pmatrix} 1/2 & 1/2 \\ -3/2 & 5/2 \end{pmatrix}.$$

(a) What are the eigenvalues of A?

(b) Compute A^{-1}.

(c) What are the eigenvalues of A^{-1}?