Math 124: Fall 2016
Practice for Exam 2

NAME:

Time: **1 hour and 15 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Problem	Value	Score
1	3	
2	4	
3	4	
4	6	
5	6	
6	5	
7	5	
8	5	
9	12	
TOTAL	50	

Problem 1 : (3 points)

a) What is the dimension of $\mathcal{M}_{4\times4}$? You do not need to justify your answer.

b) There is a unique value of k such that $\mathcal{M}_{4\times4}$ is isomorphic to $\mathbb{R}^k$. What is this value of k? You do not need to justify your answer.

c) Now let m and n be two positive whole numbers. For what k is $\mathbb{R}^k$ isomorphic to $\mathcal{M}_{m\times n}$?

d) There is a unique value of k such that $\mathcal{P}_5$ is isomorphic to $\mathbb{R}^k$. What is this value of k? You do not need to justify your answer.

Problem 2 : (4 points) Consider the vector space $\mathcal{P}_2$. For each set S, determine if S is a basis for $\mathcal{P}_2$.

a) $S = \{x^2 - x + 1, 2x + 1, 2x - 1\}$

b) $S = \{x + x^2, x - x^2, x + 2x^2\}$

Problem 3 : (4 points) Let

$$\vec{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Represent $\vec{v}$ with respect to the following two bases:

a) $B_1 = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

b) $B_2 = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$

Problem 4 : (6 points) Consider the isomorphism $f\colon \mathcal{P}_1 \to \mathbb{R}^2$ with

$$a + bx \mapsto \mathrm{Rep}_{\{1,1+x\}}(a + bx).$$

Find the image of each of the following elements:

a) $3 - 2x$

b) $2 + 2x$

c) x

Problem 5 : (6 points) For each pair of a vector $\vec{v}$ and a matrix A, decide if $\vec{v}$ is in the row space of A.

a) $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{pmatrix}$

b) $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$.

Problem 6 : (5 points) Give the dimension of the solution set of the homogeneous system of linear equations

$$\begin{aligned} x + y + 2z &= 0 \\ 2x - y + z &= 0 \\ 4x + y + 5z &= 0 \end{aligned}$$

For credit you must give some mathematical justification for your answer.

Problem 7 : (5 points) Find a basis for the row space of the matrix

$$A = \begin{pmatrix} 2 & 0 & 3 & 4 \\ 0 & 1 & 1 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & -4 & -1 \end{pmatrix}.$$

Make sure to argue that you have found a basis.

Problem 8 : (5 points) Give a basis for the subspace of $\mathcal{P}_2$ given by

$$V = \{a_2x^2 + a_1x + a_0 : a_2 - 2a_1 = a_0\}.$$

Make sure to argue that you have found a basis.

Problem 9 : (12 points) Consider the subspace

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x = 3z, y = -z, z \in \mathbb{R} \right\}.$$

a) (3 points) Give a spanning set for V.

b) (3 points) Give a basis for V. Prove that you found a basis.

c) (6 points) Show that V is isomorphic to $\mathbb{R}$.
You can use the next page for your work if you run out of space here.

Please use this page if you need extra space for any problem. (On the problem page, be sure to let me know to look here, and label each problem clearly if you work on multiple problems here.)