Math 124: Fall 2016
Exam 2

NAME:

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Problem	Value	Score
1	6	
2	6	
3	6	
4	6	
5	8	
6	8	
7	10	
TOTAL	50	

Problem 1 : (6 points) For the map $f\colon \mathcal{P}_1 \to \mathbb{R}^2$ given by

$$a + bx \mapsto \begin{pmatrix} a - b \\ b \end{pmatrix},$$

find the image of the following elements:

a) $3 - 2x$

b) $2 + 2x$

c) x

Problem 2 : (6 points)

a) (2 points) Give the definition of the word basis.

b) (4 points) Is the set

$$\left\{ \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \right\}$$

a basis for the space $\mathbb{R}^3$?

Problem 3 : (6 points) Let

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Represent $\vec{v}$ with respect to the basis

$$B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}.$$

Problem 4 : (6 points) Is the vector

$$\vec{v} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

in the column space of the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}?$$

Problem 5 : (8 points) Consider the homogeneous system of linear equations

$$\begin{aligned} x_1 - 4x_2 + 3x_3 - x_4 &= 0 \\ 2x_1 - 8x_2 + 6x_3 - 2x_4 &= 0 \end{aligned}$$

What is the dimension of its solution set? Support your answer by giving a basis.

Problem 6 : (8 points) What is the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 5 & 1 & 1 \\ 6 & 4 & 3 \end{pmatrix}?$$

Please support your answer by giving the basis of a space (specify which space that is) and arguing that you have found a basis.

Problem 7 : (10 points)

a) (3 points) Give the definition of the word isomorphism.

b) (7 points) Show that the subspace given by the x-axis of $\mathbb{R}^3$ is isomorphic to $\mathbb{R}$.
(The x-axis are the points where $y = 0$ and $z = 0$.)
Please solve this problem by giving an explicit isomorphism between the x-axis of $\mathbb{R}^3$ and $\mathbb{R}$. Make sure to check that your isomorphism satisfies all of the conditions that you state in your definition in part a).

Please use this page if you need extra space for any problem. (On the problem page, be sure to let me know to look here, and label each problem clearly if you work on multiple problems here.)