Math 124: Fall 2016
Practice for Exam 1

NAME:

Time: **1 hour and 15 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Problem	Value	Score
1	15	
2	4	
3	6	
4	6	
5	5	
6	5	
7	9	
TOTAL	50	

Problem 1 : (15 points) Solve the following systems of linear equations. For full credit, check your answer when a solution exists.

a)

$$\begin{aligned} 2y + z &= 2 \\ 2x - y + z &= 0 \\ -2x - y \quad &= -1 \end{aligned}$$

b)

$$
\begin{aligned}
x - y + z \phantom{{}+w} &= 0\\
y \phantom{{}+3z} + w &= 0\\
3x - 2y + 3z + w &= 0\\
-y \phantom{{}+3z} - w &= 0
\end{aligned}
$$

c)

$$
\begin{aligned}
x + y + 2z &= 0 \\
2x - y + z &= 1 \\
4x + y + 5z &= -1
\end{aligned}
$$

Problem 2 : (4 points) Write down the 4×4 matrix whose $a_{i,j}$ entry is $(-1)^{i+j}$.

Problem 3 : (6 points) Are the following two matrices row equivalent?

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 3 & -3 \\ 1 & 1 & 5 \end{pmatrix}$$

Problem 4 : (6 points) Consider the following subsets of $\mathcal{P}_3$, the set of polynomials of degree less than or equal to two. Decide if each of the subsets is linearly independent or linearly dependent.

a) $\{-1, 4 + x^2\}$

b) $\{-x^2, 1+4x^2, 4\}$

Problem 5 : (5 points) Let S be the set of all vectors in $\mathbb{R}^3$ that are perpendicular to the vector

$$\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}.$$

Show that the set S is a vector subspace of $\mathbb{R}^3$.

Problem 6 : (5 points) Consider the following subset of $\mathbb{R}^2$:

$$\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}.$$

Is every vector in $\mathbb{R}^2$ in the span of this set?

Problem 7 : (9 points) Consider the set of real numbers $\mathbb{R}$ with vector addition given by

$$x \oplus y = x + y + 7,$$

where $\oplus$ is the new vector addition and $+$ is the usual addition of real numbers, and scalar multiplication

$$r \otimes x = rx + 7(r - 1),$$

where again all operations on the right are the usual multiplication, addition and subtraction on real numbers.
This is a vector space.

a) Show that this vector addition is commutative.

b) What is the zero vector in this vector space?

For your convenience, here are the operations in this vector space again: Addition is given by

$$x \oplus y = x + y + 7,$$

and scalar multiplication is

$$r \otimes x = rx + 7(r - 1).$$

c) Show that addition of scalars distributes over scalar multiplication, i.e. that $(r+s) \otimes x = r \otimes x \oplus s \otimes x$.