Math 124: Fall 2016
Exam 1

NAME:

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit. For each question, you **must** use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Problem	Value	Score
1	6	
2	6	
3	13	
4	6	
5	6	
6	6	
7	7	
TOTAL	50	

Problem 1 : (6 points) Give the reduced echelon form of the following matrix:

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ -1 & -3 & -3 \end{pmatrix}$$

Problem 2 : (6 points)
Find all value(s) k such that the two vectors

$$\begin{pmatrix} k \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

are perpendicular.

Problem 3 : (13 points)

a) (5 points) Solve the following system of linear equations:

$$\begin{aligned} x \qquad - \; z \qquad &= 1 \\ y + 2z - w &= 3 \\ x + 2y + 3z - w &= 7 \end{aligned}$$

b) (4 points) Now solve this system of linear equations.
Hint: This is the homogeneous system associated to the system in part a).

$$\begin{aligned} x \quad\quad - \ z \quad\quad &= 0 \\ y + 2z - w &= 0 \\ x + 2y + 3z - w &= 0 \end{aligned}$$

c) (4 points) Check your answer to part a). (This should look like the solutions to Homework 1 or the solutions to the Practice Exam 1.)
For your convenience, the system you had to solve was:

$$\begin{aligned} x \qquad - z \qquad &= 1 \\ y + 2z - w &= 3 \\ x + 2y + 3z - w &= 7 \end{aligned}$$

Problem 4 : (6 points) Consider the set

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \right\}.$$

Is this set linearly dependent or linearly independent?

Problem 5 : (6 points) Is the vector

$$\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

in the set

$$\text{Span}\left\{ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}?$$

Problem 6 : (6 points) Give a set that spans the subspace

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 3x + 2y + z = 0 \right\}$$

of the vector space $\mathbb{R}^3$.
Hint: Pretend you are solving the linear equation $3x + 2y + z = 0$.

Problem 7 : (7 points) You have shown in suggested problems that the set of positive real numbers $\mathbb{R}^+$ is a vector space when we interpret $x \oplus y$ to mean the product of x and y (so that $2 \oplus 3 = 2 \cdot 3 = 6$), and we interpret $r \otimes x$ as the r-th power of x (in other words, $r \otimes x = x^r$).

a) (3 points) Perform the following operations, **with their meaning as above**.

   i. $3 \oplus 4$

   ii. $3 \otimes 2$

   iii. $-\frac{1}{2} \otimes 4$

b) (2 points) In this vector space, what is the zero vector? (For partial credit you may write down the axiom giving the property of the zero vector.)

c) (2 points) Prove that in this vector space, scalar multiplication distributes over vector addition, i.e. that $r \otimes (x \oplus y) = (r \otimes x) \otimes (r \oplus y)$.