Name:

To get the most of this quiz, allow yourself no more than 20 minutes to completely answer it, and do not use any notes or outside help. I will grade it if you hand it in to me on or before May 11.

Problem 1 (10 points): *Find the general solution of the system*

$$\boldsymbol{x}'(t) = \begin{bmatrix} -2 & -9 & 0 \\ 1 & 4 & 0 \\ 1 & 3 & 1 \end{bmatrix} \boldsymbol{x}(t)$$

Solution: We first find the characteristic polynomial of this matrix:

$$\begin{vmatrix} -2-\lambda & -9 & 0 \\ 1 & 4-\lambda & 0 \\ 1 & 3 & 1-\lambda \end{vmatrix} = (1-\lambda)[(-2-\lambda)(4-\lambda)+9] = (1-\lambda)(\lambda-1)(\lambda-1)$$

So we have the eigenvalue $\lambda = 1$ repeated three times. We now find the dimension of the eigenspace:

$$\begin{bmatrix} -3 & 9 & 0 \\ 1 & 3 & 0 \\ 1 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} -3 & 9 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this we see that the eigenspace is two dimensional, so we are looking for two linearly independent eigenvectors $\mathbf{v}_1$ and $\mathbf{u}_1$, and a generalized eigenvector $\mathbf{v}_2$ such that $(\mathbf{A}\text{-}\mathbf{I})^2\mathbf{v}_2$=0 and $(\mathbf{A}\text{-}\mathbf{I})\mathbf{v}_2$=$\mathbf{v}_1$.
First we find that $(\mathbf{A}\text{-}\mathbf{I})^2$=0, so we try $\mathbf{v}_2$=(1,0,0). This gives us $(\mathbf{A}\text{-}\mathbf{I})\mathbf{v}_2$=(-3,1,1)=$\mathbf{v}_1$.
We now need to find another linearly independent eigenvector $\mathbf{u}_1$. We find the eigenspace to be

$$\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s$$

Our vector $\mathbf{v}_1$ is

$$\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(in other words $t = 1$ and $s = 1$), so one of many choices for $\mathbf{u}_1$ is (0,0,1) ($t = 0$ and $s = 1$).
So the general solution is

$$\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 e^t \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} + c_3 e^t \left(\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$