MATH 130: Proof of divisibility test for 4

Theorem

A number is divisible by 4 if and only if the number formed by its last 2 digits is divisible by 4.

Proof:

Using expanded form, any whole number N can be written as

$$N = 100a + b$$

where a is some whole number and b is the number formed by the last two digits of N.

We have that $100a = 4 \cdot (25a)$ so $100a$ is divisible by 4.

By the divisibility lemma, N is divisible by 4 if and only if b, the number formed by its last 2 digits, is divisible by 4.

□

Discussion of the proof.

The proof has 3 steps:

1- using expanded form to write $N = 100a + b$, where b is the number formed by the last 2 digits of N.

2- showing that $100a$ is divisible by 4

3- using the divisibility lemma.

I will discuss each step separately.

Step 1-

① Write each of the numbers

2,838 ; 179 ; 26,344

in the form $100a + b$. Is b really the number formed by the last 2 digits? Can you really always do something like this?

② Why did we choose to write N as $100a + b$?

Why not $N = 4a + b$?

OR $N = 1000a + b$?

OR $N = 10a + b$?

Can we always write N as $4a + b$ for some numbers a and b? What about for $N = 1000a + b$ and $N = 10a + b$?

Hint: There is a theorem that we know that answers this!

③ When we write $N = 1000a + b$, what is b?
What about when we write $N = 4a + b$?

Step 2 -

① Which of the following numbers are in the form $100a$ for some whole number a:

2500 ; 230 ; 3300?

Are the numbers that are of the form $100a$ really divisible by 4?

② We say that a number is divisible by 4 if it can be written as 4 times a whole number. $100a$ is 4 times what whole number? (It might help you to think of the numbers in ① that are of the form $100a$. For each of them, write down what a is, and write down what you get when you divide the number by 4. Do you see a pattern?)

Step 3–

① When we apply the divisibility lemma to $N = 100a + b$, which number is the A from the lemma? Which number is the B from the lemma? Which number is the A+B from the lemma?

To answer this, it might help you to look back to ① in Step 1– and apply the divisibility lemma to those numbers.