Math 130: Proof of divisibility test for 4

Theorem
A number is divisible by 4 if and only if the number formed by its last 2 digits is divisible by 4.

Proof:
Using expanded form, any whole number \( N \) can be written as
\[
N = 100a + b
\]
where \( a \) is some whole number and \( b \) is the number formed by the last two digits of \( N \).

We have that \( 100a = 4 \cdot (25a) \) so 100a is divisible by 4.

By the divisibility lemma, \( N \) is divisible by 4 if and only if \( b \), the number formed by its last 2 digits, is divisible by 4.

\( \square \)
Discussion of the proof.

The proof has 3 steps:

1 - using expanded form to write $N = 100a + b$, where $b$ is the number formed by the last 2 digits of $N$.

2 - showing that $100a$ is divisible by 4.

3 - using the divisibility lemma.

I will discuss each step separately.

Step 1 -

1. Write each of the numbers
   
   2,838 ; 179 ; 26,344

   in the form $100a + b$. Is $b$ really the number formed by the last 2 digits? Can you really always do something like this?

2. Why did we choose to write $N$ as $100a + b$? Why not $N = 4a + b$?
   
   or $N = 1000a + b$?
   
   or $N = 10a + b$?

Can we always write $N$ as $4a + b$ for some numbers $a$ and $b$? What about for $N = 1000a + b$ and $N = 10a + b$?
Step 2 -

1. Which of the following numbers are in the form $100a$ for some whole number $a$:
   
   \[ 2500 \quad ; \quad 230 \quad ; \quad 3300 \] 
   
   Are the numbers that are of the form $100a$ really divisible by $4$?

2. We say that a number is divisible by $4$ if it can be written as $4$ times a whole number. $100a$ is $4$ times what whole number? (It might help you to think of the numbers in 1 that are of the form $100a$. For each of them, write down what $a$ is, and write down what you get when you divide the number by $4$. Do you see a pattern?)
Step 3-

1. When we apply the divisibility lemma to $N = 100a + b$, which number is the $A$ from the lemma? Which number is the $B$ from the lemma? Which number is the $A+B$ from the lemma? To answer this, it might help you to look back to 1 in Step 1- and apply the divisibility lemma to those numbers.