Collective Defection

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Abstract

An overwhelming number of studies in recent decades have demonstrated irrefutably that human decision-making is not rational in the neoclassical sense. Alongside, Schelling’s model of racial segregation makes clear that even simple agencies can, and regularly do, manifest system level behavior seemingly anathematic to those agencies. We adumbrate an approach to economic study straddling the gap between micro and macro, built specifically around these two pillars; (1) cognitive as opposed to rational models of agent behavior, and (2) countervalent emergence. We develop a simple agent based model as part of an imagined human cognitive-ethology, thus providing a prototype for a subtly different approach to economics and a way out of a classic quagmire regarding human nature. Our model exhibits a countervalent emergence that could explain the stability of high Gini coefficient resource distributions independent of a selfishness axiom.
Table of Contents

List of Figures .................................................. iii

1 Introduction and Theory ........................................ 1
  1.1 The Hunt for Homo-economicus ............................... 1
  1.2 Moral Grammar ............................................. 3
  1.3 Mental Grammar ............................................ 4
  1.4 Ethology to Cognitive-Ethology ............................. 5
  1.5 Economic Decisions ........................................ 7
  1.6 Rational Action as a class of reasoning, not a model of reason
ing ......................................................... 7
  1.7 Moral Grammatical Analysis of Building a Better America
one Quintile at a Time ....................................... 8
  1.8 Schelling’s Model and the “complex” relation of Micro and Macro .... 9

2 Model and Results ............................................. 10
  2.1 The Model Agency in a Cognitive-Ethological Perspective . 10
  2.2 On a Triangle ................................................ 12
  2.3 Graphs on Four Vertices: .................................... 12
  2.4 Pareto Distributions ......................................... 12
  2.5 Five Real Networks .......................................... 13
     2.5.1 Football, What’s Up With That ......................... 13
  2.6 Conclusion .................................................. 14

Bibliography ....................................................... 22
List of Figures

1  Homomorphic or Projective Model: At top are the parallel structures of language grammar, and its presumed brain organizational “organ”, and the possibly analogous moral faculty. At bottom is our organization. In place of distinct organs for moral and linguistic processing we have the idea of a mental model. Grammar is then understood as structure preservation, and morality as a question about the geometry of that structure. .......................................................... 4

2  Ethology to Cognitive-Ethological Economics. Above left is laid out the fourfold typology of cause for animal behavior descended from Tinbergen [26]. Dotted arrows indicate the identifications with analogous structures in our fourfold typology specifically for human economics. .......................................................... 5

3  Agency. When active, an agent examines their neighborhood and gives to those with less than they. First to the “poorest” amongst their neighbors, until the point when giving more would sacrifice an order relation. In (a), the central node begins by giving to the poorest of its neighbors. In (b), having given enough to the target in (a) two neighbors now have the same amount and so further giving goes to both in equal amounts. Giving continues until (c) when further giving is “blocked” by orange equality. .......................................................... 11

4  Phase space of the model on a triangle. Since the model is conservative, the three dimensional phase space can be collapsed without loss of information to an equilateral triangle. Given symmetries of the system, we need only track the images of the dotted portion of the border under the maps. On the left is the asynchronous update case, on the the right is the synchronous form of the model. Stylization of 1000 data points. Broken arrows correspond to asymptotic, normal arrows to single time-steps. .......................................................... 15

5  Posets by Containments. On the left is the partially ordered set of connected subgraphs of $k_4$ with containments given by arrows. On the right are the stable manifolds corresponding to those connected graphs. Red arrows correspond to inclusions present in the original lattice no longer present in the partially ordered set of manifolds, whereas black arrows correspond to those preserved under the map. This view suggests an interesting task might be a typology of edges by way of their effect on the stable manifold for the system. The higher dimensional analog of the dominant pair rays in the $k_3$ case are always stable. .......................................................... 16

6  In magenta at left is a histogram of the relaxation times of 100 randomly generated Pareto (top) and uniform (bottom) distributions on the Adjective-Noun network from [22] under the asynchronous form of the model. At middle in magenta are the initial and ultimate Gini coefficients of those same distributions. Also at middle are the in/out Gini coefficients of those same distributions under the synchronous model. Lastly, at right in green are histograms of the relaxation times of those distributions. .......................................................... 17

7  The same organization of data as Fig. 6 but for the neural network of C. Elegans from [29,28]. .......................................................... 18
The same organization of data as Fig. 6 but for the network of American football games between Division IA colleges during regular season Fall 2000, from [11].

The same organization of data as Fig. 6 here applied to the social network of friendships between 34 members of a karate club at a US university in the 1970s., from [30].

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1 Introduction and Theory

In 2011, Norton and Arielly published [2] an analysis of their survey of ~5,000 Americans in which participants were asked to describe, by quintiles, the wealth distribution in the United States.\(^1\) Across levels of education, boundaries of political affiliation, gender, and income, participants described the wealth distribution as being much more equal than the current situation. Americans at the time had little idea of how wealth was actually distributed\(^2\).

Norton and Arielly asked participants what they thought to be the ideal wealth distribution. Americans overwhelmingly, 93\% versus 7\%, preferred the Swedish\(^3\) wealth distribution to the one they themselves described. By and large however, Americans do not vote for, or even poll positively on explicitly redistributive policies. Under the abiding theory of human economic behavior, rational choice theory, Norton and Arielly’s finding is doubly paradoxical. If economic decision making is understood as utility maximization, then Americans vote not only against self interest, but against their own stated preferences.

Admittedly, in the interest of preserving our guiding simplification for human behavior, the rational action hypothesis, we might explain away this dilemma by the observation that governance and voting are massively constrained. In this light, we could then view deviation from voting for and polling well on this documented core desire as an artifact of political institutions established with degrees of freedom orthogonal to the central will uncovered. It is however among the hypotheses of this document, that there exists a better explanation.

1.1 The Hunt for Homo-economicus

In 2005 Henrich et al [15] used the famous Ultimatum Game first described in [13] to explore economic behavior in 15 different small scale societies. The Ultimatum Game as played in [15] functions as follows: the experimenter offers a first player an amount of money contingent on the first player making a second player an offer of some portion of that amount. If the

\(^1\)It may well be that the liberation of the wealth distribution question from familiar categorizations like class or the framework of deserving to keep what you earn, is what enabled participants to express preferences that would in usual American political discourse have branded them left wing extremists.

\(^2\)the Occupy(#occupy) movement has very likely changed our knowledge of the situation.

\(^3\)The provenance of the wealth distribution was not disclosed to participants.
second player accepts that offer, the money is distributed as the two players have agreed. If the offer is rejected neither player receives anything.

In playing this game across many societies, initial offers and rejection cutoffs varied widely. Most importantly however, in the studied societies game theoretic predictions based on assumptions of ignorance, anonymity, and a selfishness axiom were always violated. In small scale societies the assumptions of ignorance, and anonymity might very reasonably fail to hold, as such we might think this result unsurprising. To control for the inoperability of the assumption of ignorance the authors used rejection data to create a model member of each culture against whom we assume the proposer imagines themselves to be playing. Even with this change however, the results do not conform: humans do not simply maximize individual utility.

We might in the light of such obvious failures despair, and find ourselves lost in a stupefying catalog of cognitive deviations, anchoring, artificial discounting of the future value, etc. Deviation however is an unjust framing for the surface phenomena of human decision-making, seeing as in no way do we fail to be human, and as McCubbins et al. [19] note,

“Humans are able to solve many tasks that are quite difficult. Like vision, taste and smell, human intelligence and behavior are varied and flexible, creating an enormous diversity of beliefs and choices, but the models that we use to predict behavior do not and cannot capture this diversity.”

It is then the question of around which characters of cognition are we to build our model. Of the closing analyses to Henrich et al.’s work is the hope that a model of economic decision making might be built:

“from a set of basic human psychological mechanisms involving fairness and resource distribution...”

It is this explicitly moral framing regarding an abstract concept like fairness that suggests where we might look.
1.2 Moral Grammar

John Mikhail’s Ph.D. thesis, and his subsequent book [21] are minimally a renovation of John Rawls “linguistic analogy,” [23] for moral cognition. Like language, moral cognition is a sphere in which humans easily make quick decisions about the permissibility of hypothetical actions in novel situations, yet are often at a loss at a conscious level to why we feel these distinctions.

Among other investigations, Mikhail considered the famous trolley problems of ethics. Is it right to push a man onto the tracks, which results in his death, in order to save the passengers of a trolley? Is it right to let him be crushed in order to save those passengers were he already prone on the tracks? Objectively, meant in the same way as it would be invoked in the analysis of utility, the situations are the same. Either a single man, or the multiple passengers of a trolley will die. The situations are in a very real way, the same.

The moral act, the act described as moral by most participants, however differentiates the two scenarios [20]. If already prone, the majority hold it moral to let the man stop the train with his body. In the other, only a tenth hold it right push him onto the tracks. The distinction between these scenarios is the issue of intuitive causality, and deep prohibitions against battery and murder. We find the same distinction revealed by asking

“Is it right to let a man die if it will save five?”

vs.

“Is it right to kill a man to save five?”

With the same calculus, but a different blend implying different conceptual causalities, the geometry of the mental model of a dynamic system, we get different behaviors.

We preserve this linguistic analogy for moral decision making, and in fact extend it (see Fig. 1).
Figure 1: *Homomorphic* or Projective Model: At top are the parallel structures of language grammar, and its presumed brain organizational “organ”, and the possibly analogous moral faculty. At bottom is our organization. In place of distinct organs for moral and linguistic processing we have the idea of a mental model. Grammar is then understood as structure preservation, and morality as a question about the geometry of that structure.

### 1.3 Mental Grammar

Many aspects of human behavior appear to have grammars. Our position, we believe shared with Faucconier [8], Turner [27], and Jackendoff [16], is that these are the projections of the architecture of human thinking. Wherever there is an apparent generative structure, it is but a shadow of the generative structure for the central phenomena of human life, the mind.

Whatever universal linguistic grammar there may be is to be understood as the relatively paucity of ways that human thought may be mapped homomorphically into serial communication.\(^4\) Faucconier and Turner at an accessible level in [27], and in individual other texts [8,7], develop a model of human reasoning built on mental spaces, maps between them, and thereby blends. To each mental space is attached a force dynamics, a way things go. It is in the assemblage of these local dynamics that intuitive cause exists; hence the posets Mikhail extracts. It is the goal of the program to build models of economic activity along these lines.

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\(^4\)For readers interested in purely linguistic aspects of this claim, await forthcoming work Williams, Lessard et al. on power law distributions and evolved serial communication mechanisms.
1.4 Ethology to Cognitive-Ethology

Niko Tinbergen, who along with Karl von Frisch, and Konrad Lorenz won the Nobel Prize in 1973, is credited with the fourfold program [26] for the integrative analysis of animal behavior usually meant by the term Ethology [26]. Tinbergen argued that no behavior is adequately explained without telling four stories:

1. of *mechanism*: cause in the organism;

2. of *survival value*: the ultimate cause or functions of the behavior;

3. of *ontogeny*: The growth of the faculty for this behavior in the life of this organism; and

4. of *phylogeny*: The evolutionary history of this behavior.

Now we do not exactly follow Tinbergen’s challenge, as is implied by the heading. His mechanism is exactly that, and ours operates in the arena cleared by the Cognitivist revolution,
hence a Cognitive-Ethology. The manner in which we use the term is quite specific, and further modified by our application of this meta-theory to economics (see Fig. 2).

The defense of those two identifications is quite quick. The deep reason for appeals to immediately chemical or electrical models of mechanism, their structural necessity, is that they exhibit adequate regularity to make predictions, and do not hide homunculi (for clarification regarding language and claim see [6]). The cognitive revolution however is at core the realization that the supposedly higher and thereby less defined structures of cognition, of course included here is the full hierarchy of brain processes, actually exhibit more than adequate regularity to exist as base entities in an explanation of phenomena. As such, a search for direct stimulus-response relations may be avoided, and replaced with stimulus-cognition-response models.

Despite lengthening the chain we find simpler explanations. The apparent complexity of a problem in science may well be an artifact of an inappropriate theory for its description, as much as any intrinsic baroque property. For example, in laying out this program for the study of economics, we take the heft of their catalog of deviations as evidence against naive bounded rationality theories. The substitution of cognition for mechanism should not be seen as contentious. The relation between survival value and utility as used in the economic literature is clear.

Our casting of economic decision making not in terms of specific social actions, but in terms of an underlying cognitive faculty, the mental grammatical hypothesis, is what admits it into this paradigm. While we do not yet have elaborate ontogenetic and phylogenetic stories, we do place those tasks into the realm of the scientifically practicable, by identifying it with the larger task of studying the evolution of cognition. We can build a theory of economic decision making just as that capacity in us was grown, by repurposing and extending the existing mental capacities of Homo Sapiens. Moreover, we can examine the history of of economic reasoning by looking at the other projections of evolving cognitive capacities, with which it must be consistent.
1.5 Economic Decisions

Human decision making is not optimization with exact reference to structures in the world. Instead, where optimization is conceptually applicable, which does describe many economic situations, we do it with respect to internal models. To the hard scientist usually entitled to an easy separation of phenomena and theory, this might seem an unnecessary caveat, but this is precisely the character of human reason that Mikhail, Henrich, Faucconier, Turner, and we contend. We propose that all human economic decisions are best understood as picking a moral option that achieves some preference. Is morality not clearly the structure behind “what should I do?” In this view rational choice theory appears too, not itself a model of decision making, but as a description of moral reasoning in an anchored blend involving self preservation.

1.6 Rational Action as a class of reasoning, not a model of reasoning

John Nash himself said

“I realize that what I had said at some time may have over-emphasized rationality or some type of thinking and I don’t want to overemphasize rational thinking on the part of humans...that is my enlightenment...” [4].

On the meta-theoretical grounds of cognitive-ethology, any argument that human decision making fits the rational model is to confuse “personal motivations and individual cognition with the ultimate “logic” of selection.” Evolution is far too messy for that. Darwin [5] writes

“Nor ought we to marvel if all the contrivances in nature be not, as far as we can judge, absolutely perfect; and if some of them be abhorrent to our ideas of fitness. We need not marvel at the sting of the bee causing the bee’s own death; at drones being produced in such vast numbers for one single act, and being then slaughtered by their sterile sisters; at the astonishing waste of pollen by our fir-trees; at the instinctive hatred of the queen bee for her own fertile daughters; at ichneumonidae feeding within the live bodies of caterpillars; and
at other such cases. The wonder indeed is, on the theory of natural selection, that more cases of the want of absolute perfection have not been observed."

We should not spend our time finding the secret rationality in all forms of human action, seeking some Ptolemaic imagining in which our decision making is optimal. Instead we need to explain why and when rational action is compelling as a model.

Many legal codes hold that self-defense is legitimate justification for otherwise illegal action. The effect of the self preservation frame on the conception of moral action is quite clear. How often have we heard as total apology some variant of “a man’s gotta eat.” Self defense and selfishness, an admittedly pejorative term here meaning only self-oriented utility maximization, are different things, however one may build one from the other.

Self defense, and anticipation, the anchoring of a model to a particular expectation, can turn an agent’s will to self preservation into a selfishness. Collective delusion about payoffs may create selfish behavior from the simple desire not to die.

1.7 Moral Grammatical Analysis of Building a Better America one Quintile at a Time

In closing Norton and Arielly [2] write

“Given the consensus among disparate groups on the gap between an ideal distribution of wealth and the actual level of wealth inequality, why are more Americans, especially those with low income, not advocating for greater redistribution of wealth? First, our results demonstrate that Americans appear to drastically underestimate the current level of wealth inequality, suggesting they may simply be unaware of the gap. Second, just as people have erroneous beliefs about the actual level of wealth inequality, they may also hold overly optimistic beliefs about opportunities for social mobility in the United States (Benabou & Ok, 2001; Charles & Hurst, 2003; Keister, 2005), beliefs which in turn may drive support for unequal distributions of wealth. Third, despite the fact that conservatives and liberals in our sample agree that the current level of inequality is far
from ideal, public disagreements about the causes of that inequality may drown
out this consensus (Alesina & Angeletos, 2005; Piketty, 1995). Finally, and more
broadly, Americans exhibit a general disconnect between their attitudes toward
economic inequality and their self-interest and public policy preferences (Bartels,
2005; Fong, 2001), suggesting that even given increased awareness of the gap be-
tween ideal and actual wealth distributions, Americans may remain unlikely to
advocate for policies that would narrow this gap.”

Americans do not vote for redistribution, but when asked about wealth distributions by
quintiles, they vote overwhelmingly for precisely that. We propose that it is specifically the
alienness of quintiles which allows this to happen. This unfamiliar, impersonal language
liberates from, or fails to embed the topic of wealth distribution into the mental model in
which it is so often found thereby allowing for a deep preference to be revealed.

Low income persons in particular, but most Americans even, may be using essentially
personal\(^5\) models for their analysis. In these models, the prohibition against battery is lifted
to the expanded self, and thereby prohibits theft. The act of redistribution is then considered
immediately as theft. It is not the fantasy “I will be rich someday” that makes redistributive
policy unpalatable, though studies do indicate that this delusion is widespread\(^6\), it is instead
the direct identification of wealth and person, a lack of resolution in the folk theory, that
makes it \textit{immoral}.

1.8 Schelling’s Model and the “complex” relation of Micro and Macro

At least since Schelling’s model [25] of racial segregation, we have been aware that rela-
tively ambivalent agents could collectively manifest total segregation. Given the apparent
concern for fairness, especially as it turns out in market oriented societies, could we use this

\(^5\)Models wherein agents are individual human-like entities thereby having a moral “charge”.
\(^6\)The character of this observation as delusion is considered suspect by some, including the author. It
may well be that people do not play the lottery thinking they will win, but instead the play the lottery
because it feels good for just a moment to imagine that one might. Human thinking in its relation to the
world is not a lexical identification, a one to one map of representation and object, instead it is something
far more nuanced.
phenomenon that we might call countervalent emergence to account for this?

As we have claimed before, but have yet to justify, this re-conception of micro-economic theory is not a hopeless “complexification” of the issues, rendering mathematical formalism impossible. Instead, we use this new conception of decision making in logistic questions to provide an explanation for the paradoxical coexistence of fair, concerned agents and stable inequality, by way of just such an emergence.

2 Model and Results

2.1 The Model Agency in a Cognitive-Ethological Perspective

We begin with the premise that inter-agent logistic, hence economic interactions are fairness-minded, seen as moral acts of optimization of some utility function, here attempting to equidistribute a resource. Any actor would like to make the situation more equal, but will not do so if the available action breaks currently existing equalities experienced by the actor relative to its neighbors. To do so would to go against the moral principal of self preservation.

An active agent at a given time step will distribute amongst their poorer neighbors what they (the active agent) have in excess of the neighbor who has immediately less than they.

7 We build agent-behavior as local, and Markovian, thus the model can be run under any update model.8

Importantly, we find many classes of distributions away from the equidistributed state are stable. To use the language of the prisoner’s dilemma [9], we describe how we may collectively defect (see Figure 3).

7 No irony is lost on the author that the failures of a greedy algorithm to find a globally minimal structure are being used to model the non-necessity of greed as a priori character of human beings.

8 For future research the commutation relations of the update operations for individual agents are worthy of study. Connections to the theory of braids and non-commutative geometry are readily made.
Figure 3: Agency. When active, an agent examines their neighborhood and gives to those with less than they. First to the “poorest” amongst their neighbors, until the point when giving more would sacrifice an order relation. In (a), the central node begins by giving to the poorest of its neighbors. In (b), having given enough to the target in (a) two neighbors now have the same amount and so further giving goes to both in equal amounts. Giving continues until (c) when further giving is “blocked” by orange equality.
2.2 On a Triangle

Within the hierarchy of graphs the “first” topologically non trivial connected graph is the triangle $K_3$. We use it as an example structure on which the model might be played. The model conserves the resource globally, hence for $n$ nodes, the phase space naturally fills the $n-1$ simplex with barycentric coordinates. Further, the image of any ray out from the center in the simplex under the map is determined by the image of its intersubsection with the boundary.

Under simultaneous update, frustration away from equidistribution on $k_3$ is vanishingly unlikely. Heuristically, we might describe the situation as “overcompensation preserves system flexibility.” Under asynchronous sequential update however, the model tends to frustration on the dominant pair surfaces, the collection of all points in phase space where two nodes are matched, having more than the third node. Although the stable manifolds are the same for both update models, the exploration of these surfaces differ widely between synchronous and asynchronous models.

2.3 Graphs on Four Vertices:

On four vertices the possible connected graphs are more varied. Consequently the stable manifolds for the resource distribution game are as well (See Fig 5).

2.4 Pareto Distributions

A major topic of recent scientific inquiry has been power-law size distributions. Such distributions, characterized by some few agents possessing the bulk of the total resource in the system have interesting, if simple behavior in this model. Any agent possessing an extreme excess will flatten their neighborhood, after which no further flow out from this neighborhood will occur. Sufficiently large inequalities cannot flatten globally. Euphemistically, the game transitions from a monarchy to an oligarchy and stops there.
2.5 Five Real Networks

We seeded five real world networks, the adjective-noun network from [22] the neural network of C. Elegans [29,28], the network of college football games [11], the famous karate club network from [30], and the co-occurrence network of characters in Les Miserables [18]. Studying the preservation of wealth or resource distribution inequality on networks of arbitrary size requires a general map from phase space to some simpler domain, This map must also capture what we mean intuitively by inequality. Here we use the Gini coefficient [10].

The Gini coefficient takes values on the interval $[0, 1]$, with the value zero corresponding to total equality, and the value one corresponding to the situation in which a single agent possesses all of the given resource. Asynchronous update on these networks exhibits an excellent linear fit plotting initial Gini coefficient to ultimate Gini coefficient. Asynchronous update always preserve inequality better than the synchronous case. (See Fig. 7-11). Except for the football network, dealt with in the next subsection, no network allows for decay to equidistribution under either model.

2.5.1 Football, What’s Up With That

We observe that on the football network, simultaneous update does in fact result in equidistribution almost every time. Size alone cannot account for this as it is over twice the size of the karate network which does not exhibit this phenomenon. Similarly, average degree alone does not account as the distinctly non-social neural network of C.Elegans beats the football network there. We believe the reason lies in the combination of modularity and average degree, an intuitive “social health” of the network. High average degree allows initially dominant nodes to decay far enough into their neighborhoods so that surrounding noise, from other co-occurring but smaller transfers, can preserve the flexibility of the neighborhood. It is important to note however that under asynchronous update this does not seem to have any such effect because the noise aspect finds no analog, and results across networks for decay constant were much more alike.
2.6 Conclusion

In closing we make two points. First that a synthetic view of human cognition as the integration, loosely speaking, of the pullbacks of the various domains of human action morality, language, etc., provides an adequate framework for testable models of human economic decision making that within economics proper it may be difficult to find. Furthermore these models can be followed through to explore carefully the relation between micro-economics and macro-economics. Lastly the non-trivialities of this map may further be found to explain many of apparent aberrances that naive maps up or down, micro to macro and *vice versa*, manufacture. In doing so, we make testable many difficult ideas, among them that apparent selfishness is an artifact not of human nature identified with the individual, but a socially contingent phenomenon built from alienation.
Figure 4: Phase space of the model on a triangle. Since the model is conservative, the three dimensional phase space can be collapsed without loss of information to an equilateral triangle. Given symmetries of the system, we need only track the images of the dotted portion of the border under the maps. On the left is the asynchronous update case, on the right is the synchronous form of the model. Stylization of 1000 data points. Broken arrows correspond to asymptotic, normal arrows to single time-steps.
Figure 5: Posets by Containments. On the left is the partially ordered set of connected subgraphs of $k_4$ with containments given by arrows. On the right are the stable manifolds corresponding to those connected graphs. Red arrows correspond to inclusions present in the original lattice no longer present in the partially ordered set of manifolds, whereas black arrows correspond to those preserved under the map. This view suggests an interesting task might be a typology of edges by way of their effect on the stable manifold for the system. The higher dimensional analog of the dominant pair rays in the $k_3$ case are always stable.
Figure 6: In magenta at left is a histogram of the relaxation times of 100 randomly generated Pareto (top) and uniform (bottom) distributions on the Adjective-Noun network from [22] under the asynchronous form of the model. At middle in magenta are the initial and ultimate Gini coefficients of those same distributions. Also at middle are the in/out Gini coefficients of those same distributions under the synchronous model. Lastly, at right in green are histograms of the relaxation times of those distributions.
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