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SUPPLEMENTARY MATERIALS

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GROUP DYNAMICS

Network science on belief system dynamics under logic constraints

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Breakthroughs have been made in algorithmic approaches to understanding how individuals in a group influence each other to reach a consensus. However, what happens to the group consensus if it depends on several statements, one of which is proven false? Here, we show how the existence of logical constraints on beliefs affect the collective convergence to a shared belief system and, in contrast, how an idiosyncratic set of arbitrarily linked beliefs held by a few may become held by many.

onverse [(1), p. 207] defined a belief system as "a configuration of ideas and attitudes in which the elements are bound together by some form of constraint or functional interdependence." The existence of belief systems is widely accepted and a subject of interest in the scientific community (2-4), but there are still unresolved puzzles. According to cognitive consistency theory, inconsistent beliefs cause tension that individuals seek to resolve (5, 6). Thus, if an individual's certainty of belief on the truth of one statement is altered, the alteration may propagate changes of the individual's certainties of beliefs on the truth of other statements. Individual-level, independent adjustments of certainties of belief (7-14) do not suffice to explain the existence of shared beliefs in a population of individuals. Some additional, natural, social control and coordination mechanism is required. Public dispute on global warming is a prominent case in which individuals have varying certainties of belief on the truth values of a logically interdependent set of statements, which has implications for reaching a

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conclusion that collective action is required to mitigate global warming. Debates in economics on appropriate macroeconomic policy, and debates in politics on acceptable legislation, are also examples of interpersonal influences modifying individuals' certainties of belief on multiple interdependent statements. A critical open problem is the theoretical integration of theory on cognitive consistency and theory on interpersonal influence systems. We report a generalization of the Friedkin-Johnsen model (15-17) that achieves this integration. When individuals' beliefs on multiple statements are being influenced, the Friedkin-Johnsen model assumes that a change of belief on one statement does not affect beliefs on other statements. We develop and apply a more realistic model on the dynamics of belief systems in which individuals' certainties of belief on a set of interdependent true or false statements are being changed by network mechanisms of interpersonal influence.

A shared logic constraint structure on a set of truth statements (e.g., if X is true, then Y and Z are true) does not imply belief consensus. It will polarize a population into two opposing ideological factions when high certainty of belief on one central statement implies high certainties of belief on all other statements, and low certainty of belief on that central statement implies low certainties of belief on all other statements. One faction accepts the premise of the central statement and thus accepts all the other statements as true; the other rejects the premise of the central statement and thus rejects all the other statements as false. How can we better understand the dynamics of belief systems in which individuals' certainties of belief are modified by network mechanisms of interpersonal influence toward a consensus on a set of interdependent beliefs?

An analyzable problem on belief system dynamics can be posed as follows. Let us start from a state of heterogeneity in a population of individuals (i) with various levels of certainty of belief on the truth values of two or more truth statements and (ii) with a common set of logical constraints that associate these statements. In this population, levels of certainty of belief about one statement are associated with levels of certainty of belief about another statement and, more generally, an individual's level of certainty of belief about one statement is some mixture of that individual's certainty of beliefs about other statements. Let each individual's certainty about each statement be subject to disturbance. Cognitive consistency theory posits that the disturbance will cause a within-individual change that recalibrates their certainties of beliefs to achieve consistency. Let each individual in this population be embedded in a social network that allows interpersonal influences on individuals' beliefs. With such a network, cognitive consistency effects are now competing with effects of other individuals' displayed beliefs.

In our model (Fig. 1), individual nodes have different certainties of belief on multiple truth statements, which may be changed through their interactions with others. The nodes may vary in their levels of closure-openness to influence. Each node's integration of their own and others' displayed certainties of belief may be subject to logical interdependencies among statements. These interdependencies can be expressed as a matrix of logic constraints.

The dynamics of this *n*-individual belief system on m truth statements is defined by the tensor matrix equation (18)

$$\mathbf{X}(k+1) = \mathbf{AWX}(k)\mathbf{C}^{T} + (\mathbf{I} - \mathbf{A})\mathbf{X}(0)$$

where k = 0, 1, ... The $\mathbf{X}(0)$ is a $n \times m$ matrix of n individuals and m truth statements with truth values (true or false) on which individuals have heterogeneous certainties of belief in the [0, 1] interval, such that $x_{ij} = 0.50$ corresponds to an i with maximum uncertainty on the truth value of statement j of the mstatements; $x_{ij} = 1$ corresponds to an i with

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maximum certainty that the truth value of statement *j* is true; and $x_{ii} = 0$ corresponds to an i with maximum certainty that the truth value of statement i is false. A simple one-toone correspondence (bijective function) exists between individuals' emotive attitudes toward statements and their certainties of belief on statements. These two forms of evaluative orientation to truth statements are not the same, but are naturally associated. That is, the stronger i's positive attitude toward a statement, the greater *i*'s certainty of belief that a statement is true, and the stronger i's negative attitude toward a statement, the greater i's certainty of belief that a statement is false. The C is, in the simplest case, a $m \times m$ matrix of interdependencies among the m truth statements $\left(0 \le c_{ij} \le 1 \ \forall ij, \sum_{j=1}^{m} c_{ij} = 1 \ \forall i\right)$. The **W** is a $n \times n$ matrix of weights, each row *i* of which corresponds to individual i's allocations of weights to the n individuals' displayed certainties of belief $(0 \le w_{ij} \le 1 \forall ij, \sum_{j=1}^{n} w_{ij} = 1 \forall i).$ The **A** is a $n \times n$ diagonal matrix with values

 $(0 \le a_{ii} \le 1, a_{ii} = 1 - w_{ii} \ \forall i)$ that correspond to individual i's level of openness (maximally 1) or closure (minimally 0) to interpersonal influences on *i*'s certainties of belief. The supplementary materials contain a deeper introduction to the mathematical analysis of the model and references to publications with tests of the predictions of the interpersonal influence mechanism that it assumes.

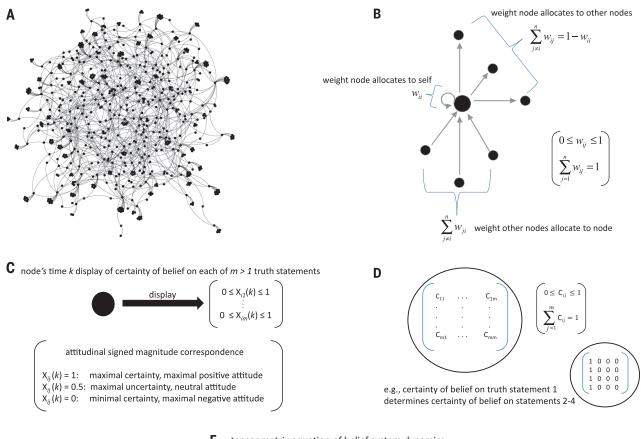
We next show how the 1992-2003 fluctuations of the U.S. population's certainties of belief on truth statements involved in the decision to invade Iraq may be understood. During the period 1992 to 2002, U.S. public opinion polls indicated that a slight majority supported an invasion of Iraq, and that a strong majority favored waiting for the conclusion of UN inspections on the status of Iraq's weapons of mass destruction (19-22). In January 2003, President Bush's State of the Union address included a threat assessment of Iraq's weapons and intentions. He stated, "We will consult, but let there be no misunderstanding: If Saddam Hussein does not fully disarm for the safety of our people, and for the peace of the world, we will lead a coalition to disarm him." In February 2003, Colin Powell, the highly respected U.S. Secretary of State and former Chairman of the Joint Chiefs of Staff, spoke to the UN Security Council. Polling just prior to the speech indicated that a strong majority of the public viewed Powell's forthcoming speech as an important factor in settling their minds about an attack on Iraq (23). The speech (24) presented a logic structure on three truth statements:

Statement 1. Saddam Hussein has a stockpile of weapons of mass destruction.

Statement 2. Saddam Hussein's weapons of mass destruction are real and present dangers to the region and to the world.

Statement 3. A preemptive invasion of Iraq would be a just war.

It was a logic structure in which high certainty of belief on statement 1 implies high certainty of belief on statements 2 and 3. Powell concluded his speech with the words, "We must not shrink from whatever is ahead of us. We must not fail in our duty and our responsibility to the citizens of the countries that are represented by this body." In March 2003, the U.S. government announced that "diplomacy has



Е tensor matrix equation of belief system dynamics

$$\mathbf{X}(k+1) = \mathbf{AWX}(k)\mathbf{C}^{T} + (\mathbf{I} - \mathbf{A})\mathbf{X}(0), \ k = 0, 1, \dots$$

 $\left(\mathbf{A}_{n \times n}, a_{ii} = 1 - w_{ii} \forall i, a_{ij} = 0 \forall i \neq j\right)$

Fig. 1. Model of belief system structure and dynamics. (A) An influence network, (B) with nodes that are allocators of influence to other nodes, (C) whose certainties of belief on multiple truth statements are influenceable, (D) under the condition of logical interdependencies among the statements, (E) based on a weighted averaging mechanism that is updating each node's certainty of belief on each truth statement.

failed" and that it would proceed without UN Security Council approval with a "coalition of the willing." President Bush spoke to the American public and announced Operation Iraqi Freedom. He stated, "The people of the United States and our friends and allies will not live at the mercy of an outlaw regime that threatens the peace with weapons of mass murder." The 2003 invasion of Iraq began a few days later. In the immediate March-May aftermath of the invasion, polling indicated a surge to strong majority support of the preemptive invasion. With the failure to find any evidence of weapons of mass destruction in Iraq, polling indicated that a strong majority of the public believed that the Iraq War was based on incorrect assumptions (25). In September 2005, Colin Powell acknowledged that his UN speech was based on flawed intelligence reports.

Two events underlie the fluctuation of public opinion on the war. The first event set up a logic structure and a conclusion. If statement 1 is true, then statements 2 and 3 are true, and the available evidence indicates that statement 1 is without doubt true. The ensuing public discourse elevated the belief that an invasion was justified. The invasion occurred. The second event, no weapons of mass destruction were found, altered the conclusion of the logic structure. The ensuing public discourse elevated the belief that the invasion was unjustified. For if statement 1 is false, then statements 2 and 3 are also false, and the available evidence indicates that statement 1 is without doubt false.

Applying the Fig. 1 model, consider a population that (i) is attentive to Powell's UN speech logic structure, (ii) maximally open to interpersonal influence, (iii) accepts its logic structure, and (iv) is connected in a regular influence network structure that allows direct or indirect flows of influence from every individual i to every individual j of the population. If this population has a high certainty on statement 1, then the belief system dynamics will generate a consensus that a preemptive invasion is a just war for any distribution of certainties of belief on statements 2 and 3. And if an event occurs that proves statement 1 false, then the population's certainty belief on all three statements will be dramatically lowered.

Figure 2 illustrates the different results of belief dynamics with and without the logic structure in which statements 2 and 3 certainties of belief depend on statement 1 certainty of belief. The colored lines distinguish individual temporal trajectories of belief on each of the three statements. Figure 2A is a population without such a logic structure. Its distinctive distributions of certainty of belief on the three statements are independent. The population converges to consensus on each statement. A high-certainty consensus is reached on the truth of statement 1. A consensus is reached that entails near maximum uncertainty on the truth of statement 2. A high-certainty consensus is reached that statement 3 is false. Figure 2B is a population with the logic structure. Its distinctive distributions

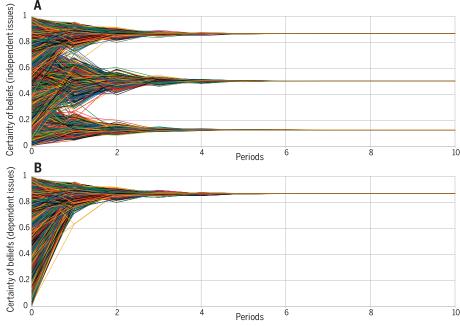


Fig. 2. Belief heterogeneity on three truth statements is reduced by the interpersonal influence system. The form of the reduction depends on the presence or absence of a belief constraint structure. (A) The three statements of belief are independent

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(B) Certainties of belief on statement 1 constrain certainties of belief on statements 2 and 3

$$\bm{C} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

The supplementary materials provide the technical details on this figure.

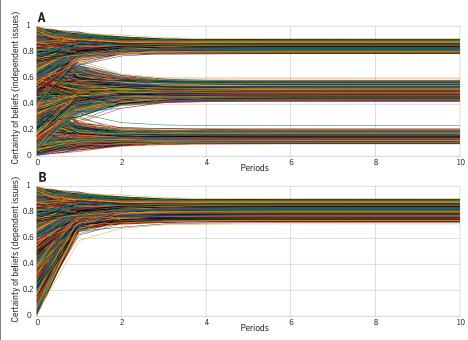


Fig. 3. When people are more wedded to their initial beliefs, the interpersonal influence system reduces, but does not eliminate, belief heterogeneity on the three statements. The form of the reduction depends on the presence or absence of a belief constraint structure. (A) The three statements of belief are independent. (B) Certainties of belief on statement 1 constrain certainties of belief on statements 2 and 3.

of certainty of belief on the three statements are interdependent. A high-certainty consensus is reached that all three statements are true.

Figure 3 relaxes the assumption that individuals' levels of openness to interpersonal influence are all maximal and introduces a level of closure to interpersonal influence that modestly anchors individuals on their initial beliefs. In contrast to the belief trajectories of Fig. 2, the effect of such anchorage is a maintained heterogeneity of beliefs under the same conditions of initial belief and network connectivity that generated the consensus results of Fig. 2. It can be shown that with more markedly heterogeneous levels of closure to influence, the evolution of the belief system is disorganized.

Figures 2B and 3B elucidate the conditions under which substantial shifts of public opinion occur when a highly influenceable public accepts a government's logic structure on truth statements. The next three figures illustrate alternate realizations of belief system dynamics on the same truth statements.

Figure 4 considers an alternative system with the same structure as Figs. 2B and 3B, but one in which the influence system overrides the implications of the logic structure. It contains an intransigent faction of k skeptics, maximally closed interpersonal influence, with identical and uniform low confidence (0.10) on all three truth statements. All other n-k individuals are maximally open to influence. Thus, the intransigent faction is composed of individuals (i) who reject the n-k others' high certainty of belief on the truth of statement 1, (ii) who are substantially less certain on the truth of statement 2 than the n-k others, and (iii) who are as skeptical as the n-k others' on statement 3. Figure 4 shows that the temporal movement is toward a consensus of diminished confidence on statements 1 and 2. More generally, if k intransigent individuals have identical beliefs $\mathbf{z} = (z_1, z_2, z_3)$, then the beliefs of the open-minded individuals converge to the row vector $\mathbf{z}\mathbf{C}^{T}$. In Fig. 4, the beliefs z_i are uniform $z_1 = z_2 = z_3 = 0.10$, and thus the whole community reaches consensus. For nonuniform certainties of belief of the intransigent individuals $z_1 \neq z_2 \neq z_3$, in general $\mathbf{z}\mathbf{C}^T \neq \mathbf{z}$, and the final beliefs of the intransigent and openminded individuals may disagree.

Figure 5 considers an alternative system with the same initial distribution of beliefs and influ-

ence network as Fig. 2B, but one with a crosspressure logic structure in which statements 1 and 3 are independent competing determinants of the statement 2 appraisal of a real and present danger,

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0.80 & 0 & 0.20 \\ 0 & 0 & 1 \end{pmatrix}.$$

The cross-pressure coefficients c_{21} and c_{23} in row 2 of the logic structure (depending on their values) determine the direction and extent of movement of statement 2 beliefs. If the row 2 values of this logic structure were reversed, $c_{21} = 0.20$ and $c_{23} = 0.80$, then the movement of public opinion would gravitate toward a rejection of statement 2 that Saddam Hussein's weapons of mass destruction are real and present dangers to the region and to the world.

Our final illustration relaxes the assumption of a common logic structure and considers a smallscale system of n = 6 policy-makers engaged in debate on the three truth statements under the condition of two competing logic structures

$$\mathbf{C}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } \mathbf{C}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

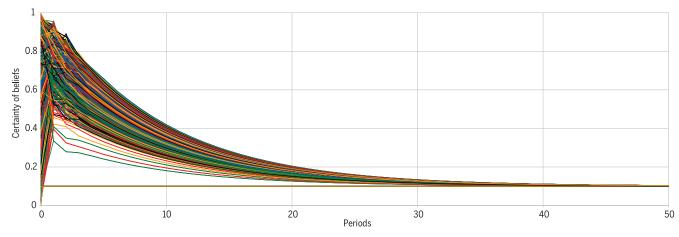


Fig. 4. A faction of intransigent skeptics overrides the belief constraint structure and generates a consensus that all three statements are false.

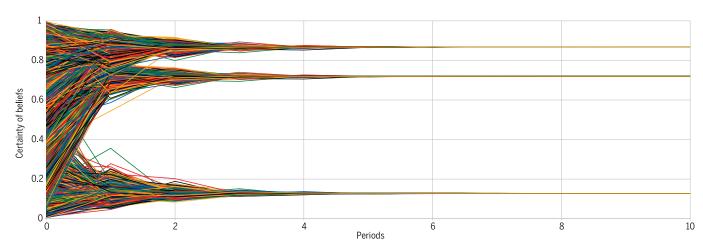


Fig. 5. Belief system dynamics with a cross-pressure logic structure in which statements 1 and 3 are independent competing determinants of the statement 2 appraisal of a real and present danger.

For three decision-makers, the position on statement 1 determines the conclusion on statements 2 and 3. For the other three decisionmakers, their conclusion on statement 3 is not determined by their position on statement 1. Let the initial beliefs of the six policy-makers be consonant with initial certainties of the larger population of citizens in which the policy group is situated,

$$\mathbf{X}(0) = egin{pmatrix} 0.96 & 0.56 & 0.16 \ 0.94 & 0.54 & 0.14 \ 0.92 & 0.52 & 0.12 \ 0.88 & 0.48 & 0.08 \ 0.86 & 0.46 & 0.06 \ 0.84 & 0.44 & 0.04 \end{pmatrix};$$

that is, each decision-maker has high certainty that statement 1 is true, uncertainty on the truth of statement 2, and high certainty that statement 3 is false, as in our previous illustrations. If all six of these decision-makers are motivated to reach consensus, and maximally open to interpersonal influence, then the belief-system dynamics are determined by the two logic structures, C_1 and C_2 , and the individuals' \mathbf{W} matrix of $i \frac{w_{ij}}{j}$ allocated weights to other individuals' displayed positions on the statements. Let this matrix be

$\mathbf{W} =$	0	0.80 0 0.80 0.80 0.80 0.80	0.20	0	0	0	
	0.50	0	0.50	0	0	0	
	0.20	0.80	0	0	0	0	
	0	0.80	0	0	0.10	0.10	,
	0	0.80	0	0.10	0	0.10	
	0	0.80	0	0.10	0.10	0	
	-						

where the block partition indicates that individuals 1 to 3 are processing information based on the \mathbf{C}_1 logic structure and individuals 4 to 6 are processing information based on the \mathbf{C}_2 logic structure. Note that the $i \underbrace{w_{ij} > 0}_{j} j$ weights are dense among individuals with the same logic structure and that individual 2 with the \mathbf{C}_1 logic structure is allocated disproportionate

weight by the five other members. The generalization of the basic equation for the competing structures is presented as eqs. 23 and 24 in the supplementary materials, and we discuss the stability in models with multiple dependency constraints in supplementary text S.2.3.3. Figure 6 shows the certainty of belief trajectories for each of the six individuals on each of three statements. With low diversity of initial opinion on each statement and an influence network in which one individual has high influence centrality, a consensus is rapidly reached that is consistent with the C_1 logic structure. With its dampened opportunity for a vigorous debate on the merits of the C_2 decoupling of statement 3 from statement 1, this group illustrates the potential hazards of the "groupthink" systems (26) that have been associated with policy-decision fiascoes.

In conclusion, truth statement interdependencies matter, and their manifestations may be diverse when individuals are embedded in an interpersonal influence system that is modifying individuals' certainties of belief. Belief system dynamics depend on the topology of truth statements' interdependencies and the topology of the influence network in which individuals are embedded. Interpersonal influence networks set up a complex system.

Individuals' certainties of belief may be elevated or dampened, beliefs about different objects may be linked, and shared belief systems may be generated. The information environment to which individuals are responding by updating their belief systems includes other individuals who are displaying their certainties of beliefs on the same truth statements. Individuals who are exposed to such social information may have heterogeneous levels of closure or openness to interpersonal influence, they may vary in who is included in the subsets of individuals whose certainties of beliefs are visible to them, and they may vary in the weights allocated to particular individuals' displayed positions. The influence network that is assembled by individuals' allocations of weights allows both direct and indirect interpersonal influences on individuals' certainties of belief on multiple truth statements.

Although possible realizations of belief system dynamics are infinite, a parsimonious, analytically tractable, general theory may cover them. Special cases include systems in which no logical interdependencies exist among truth statements, no interpersonal influences exist among individuals of the system, or no consensus is possible. The model posits two individual-level information integration processes in which a logic constraint structure is nested in an interpersonal influence process. Dynamic information environments, which include information on other individuals' certainties on multiple truth statements, continuously activate self-organizing cognitive consistency processes. If such continuous activation is a source of individual-level stress, then buffering defenses to the disturbances of the information environment may occur. Such individual defense responses include a reduced openness to interpersonal influence, a restructuring of the allocated weights to sources of information, or a flight into a local environment in which the individual is exposed only to selfconfirming information. Currently, the model does not consider such defense responses. The mathematics of the model allow a calibration of the influence network and logic constraint structures of a population; that is, an inference on what logic structure is consistent with observed trajectories of belief.

Belief system dynamics occur in both largescale populations and in small groups. Their implications are especially potent in the debates that arise in small policy groups, whose decisions affect the collective actions of governments and other organizations and, in turn, the security and welfare of numerous individuals. The hazard rate of policy fiascoes may be reduced with a more detailed attention to (i) structural features of small-group interpersonal influence systems and (ii) applications of formal rules of debate

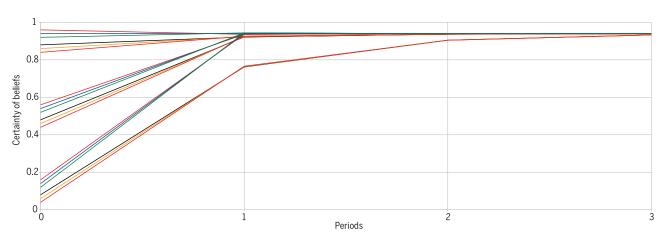


Fig. 6. Belief system dynamics in a small policy group of six individuals engaged in debate on the three truth statements under the condition of two competing logic structures. In one, the position on statement 1 determines the conclusion on statements 2 and 3. In the other, the conclusion on statement 3 is not determined by the position on statement 1. With its quick dismissal of the latter, this group illustrates the "groupthink" systems that have been associated with policy-decision fiascoes.

that can regulate these systems. The field of science on this is in its infancy.

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SUPPLEMENTARY MATERIALS

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PLANKTON DYNAMICS

Physiological and ecological drivers of early spring blooms of a coastal phytoplankter

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Climate affects the timing and magnitude of phytoplankton blooms that fuel marine food webs and influence global biogeochemical cycles. Changes in bloom timing have been detected in some cases, but the underlying mechanisms remain elusive, contributing to uncertainty in long-term predictions of climate change impacts. Here we describe a 13-year hourly time series from the New England shelf of data on the coastal phytoplankter *Synechococcus*, during which the timing of its spring bloom varied by 4 weeks. We show that multiyear trends are due to temperature-induced changes in cell division rate, with earlier blooms driven by warmer spring water temperatures. *Synechococcus* loss rates shift in tandem with division rates, suggesting a balance between growth and loss that has persisted despite phenological shifts and environmental change.

arine phytoplankton account for one-half of global primary production. Of considerable interest and concern is how climate change may affect this production. Increased temperature, ocean acidification, and altered nutrient delivery all have the potential to affect phytoplankton dynamics, including the timing and magnitude of blooms, which can dominate seasonal productivity (I, 2). There is evidence of current and ongoing changes in plankton phenology (3-5), with potentially substantial ecological consequences for marine systems (6).

Recently, there has been uncertainty about the detection of trends in phytoplankton biomass and how possible trends relate to climate change (7-9). The uncertainty arises in part from difficulties in species-level detection of phytoplankton. Many studies use bulk measurements that reflect a composite of the phytoplankton community (10). These measurements (such as chlorophyll concentration) can mask taxon-specific changes and obscure the mechanisms that govern responses to climate change. Another challenge lies in the need to observe and measure phytoplankton at appropriate time scales to elucidate those mechanisms. Ecological interactions and physiological responses of phytoplankton are rapid (on the order of minutes to hours). To adequately capture population dynamics, we must sample at this frequency, but also for extended durations because identification of seasonal, yearly, or decadal trends requires time series of these lengths.

We address this lack of temporal and taxonomic resolution for the picophytoplankter Sym*echococcus* by using observations of individual cells and their properties from an automated submersible flow cytometer, FlowCytobot (FCB) (*11*), deployed at the Martha's Vineyard Coastal Observatory (MVCO). FCB has been deployed at MVCO since 2003, with year-round observations beginning in 2007. The data consist of a 13-year time series of hourly measurements of *Synechococcus* concentration and cell properties.

At MVCO, Synechococcus concentration exhibits a strong seasonal cycle, with low concentrations in winter and early spring, followed by a two- to three-order-of-magnitude bloom event in late spring (Fig. 1A). The population fluctuates around a slowly declining trend during summer and early fall and then declines sharply in late fall. Although this classic pattern (12) is stable from year to year, we found that the timing of the spring bloom varied by up to 4 weeks within our time series, and in particular we noted a trend of earlier blooms from 2003 to 2012 (~20-day advance) and later blooms from 2013 to 2015. We quantified these shifts by determining the day of the year at which the concentration first exceeds threshold concentration levels (Fig. 2B and fig. S1). Concurrent observations of temperature (Fig. 1D) show that earlier blooms coincide with warmer spring conditions (Fig. 2A and fig. S2). For each degree increase of the mean temperature in April, the spring bloom advances 4 to 5 days. The water at MVCO has been warming (fig. S3) in a manner consistent with the multidecadal trend in this region (13). Large seasonal and interannual variations are superimposed on these warming trends.

Numerous studies have identified correlations between temperature and *Synechococcus* concentration across a range of ocean conditions (12, 14–17). In particular, there is evidence that the spring bloom begins in northeast U.S. and Canadian waters when the temperature exceeds

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Network science on belief system dynamics under logic constraints

Noah E. Friedkin, Anton V. Proskurnikov, Roberto Tempo and Sergey E. Parsegov (October 20, 2016) *Science* **354** (6310), 321-326. [doi: 10.1126/science.aag2624]

Editor's Summary

Belief system dynamics

People tend to structure their beliefs in a way that appears consistent to them. But how do some beliefs within groups persist in the face of social pressure, whereas others change and, by changing, influence a cascade of other beliefs? Friedkin *et al.* developed a model that can describe complexes of attitudes in a group that interact and change (see the Perspective by Butts). Their model revealed how the changing views of the U.S. population on the existence of weapons of mass destruction in Iraq changed their views on whether the invasion by the United States was justified.

Science, this issue p. 321; see also p. 286

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