

Robust dynamic classes revealed by measuring the response function of a social system

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We study the relaxation response of a social system after endogenous and exogenous bursts of activity using the time series of daily views for nearly 5 million videos on YouTube. We find that most activity can be described accurately as a Poisson process. However, we also find hundreds of thousands of examples in which a burst of activity is followed by an ubiquitous power-law relaxation governing the timing of views. We find that these relaxation exponents cluster into three distinct classes and allow for the classification of collective human dynamics. This is consistent with an epidemic model on a social network containing two ingredients: a power-law distribution of waiting times between cause and action and an epidemic cascade of actions becoming the cause of future actions. This model is a conceptual extension of the fluctuation-dissipation theorem to social systems [Ruelle, D (2004) *Phys Today* 57:48–53] and [Roehner BM, et al., (2004) *Int J Mod Phys C* 15:809–834], and provides a unique framework for the investigation of timing in complex systems.

complex systems | human dynamics

Uncovering rules governing collective human behavior is a difficult task because of the myriad of factors that influence an individual's decision to take action. Investigations into the timing of individual activity, as a basis for understanding more complex collective behavior, have reported statistical evidence that human actions range from random (1) to highly correlated (2). Although most of the time, the aggregated dynamics of our individual activities create seasonal trends or simple patterns, sometimes our collective action results in blockbusters, best sellers, and other large-scale trends in financial and cultural markets.

Here, we attempt to understand this nontrivial herding by investigating how the distribution of waiting times describing individuals' activity (3) is modified by the combination of interactions (4) and external influences in a social network. This is achieved by measuring the response function of a social system (5) and distinguishing whether a burst of activity was the result of a cumulative effect of small endogenous factors or, instead, the response to a large exogenous perturbation. Looking for endogenous and exogenous signatures in complex systems provides a useful framework for understanding many complex systems and has been successfully applied in several other contexts (6).

As an illustration of this distinction in a social system, consider the example of trends in queries on internet search engines (<http://trends.google.com>) in Fig. 1, which shows the remarkable differences in the dynamic response of a social network to major social events. For the "exogenous" catastrophic Asian tsunami of December 26th, 2004, we see that the social network responded suddenly. In contrast, the search activity surrounding the release of a Harry Potter movie has the more "endogenous" signature generated by word of mouth, with significant precursory growth and an almost symmetric decay of interest after the release. In both "endo" and "exo" cases, there is a significant burst of activity. However, we expect to be able to distinguish the post peak relaxation dynamics on account of the very different processes that resulted in the bursts. Furthermore, we expect the relaxation process to depend on the interest of the population because this

will influence the ease with which the activity can be spread from generation to generation.

To translate this qualitative distinction into quantitative results, we describe a model of epidemic spreading on a social network (7) and validate it with a dataset that is naturally structured to facilitate the separation of this endo/exo dichotomy. Our data consist of nearly 5 million time series of human activity collected subdaily over 8 months from the fourth most visited web site [YouTube (<http://youtube.com>), according to Alexa.com]. At the simplest level, viewing activity can occur one of three ways: randomly, exogenously (when a video is featured), or endogenously (when a video is shared). This provides us with a natural laboratory for distinguishing the effects that various impacts have and allows us to measure the social "response function."

The Model

Various factors may lead to viewing a video, which include chance, triggering from email, linking from external websites, discussion on blogs, newspapers, and television, and from social influences. The epidemic model we apply to the dynamics of viewing behavior on YouTube uses two ingredients whose interplay captures these effects.

The first ingredient is a power law distribution of waiting times describing human activity (2, 3, 8) that expresses the latent impact of these various factors by using a response function, which, on the basis of previous work (9–11), we take to be a long-memory process of the form

$$\phi(t) \sim 1/t^{1+\theta}, \quad \text{with } 0 < \theta < 1. \quad [1]$$

By definition, the memory kernel $\phi(t)$ describes the distribution of waiting times between "cause" and "action" for an individual. The cause can be any of the above mentioned factors. The action is for the individual to view the video in question after a time t since she was first subjected to the cause without any other influences between 0 and t , corresponding to a direct (or first-generation) effect. In other words, $\phi(t)$ is the "bare" memory kernel or propagator, describing the direct influence of a factor that triggers the individual to view the video in question. Here, the exponent θ is the key parameter of the theory that will be determined empirically from the data.

The second ingredient is an epidemic branching process that describes the cascade of influences on the social network. This process captures how previous attention from one individual can spread to others and become the cause that triggers their future attention (12). In a highly connected network of individuals whose interests make them susceptible to the given video content, a given factor may trigger action through a cascade of intermediate steps.

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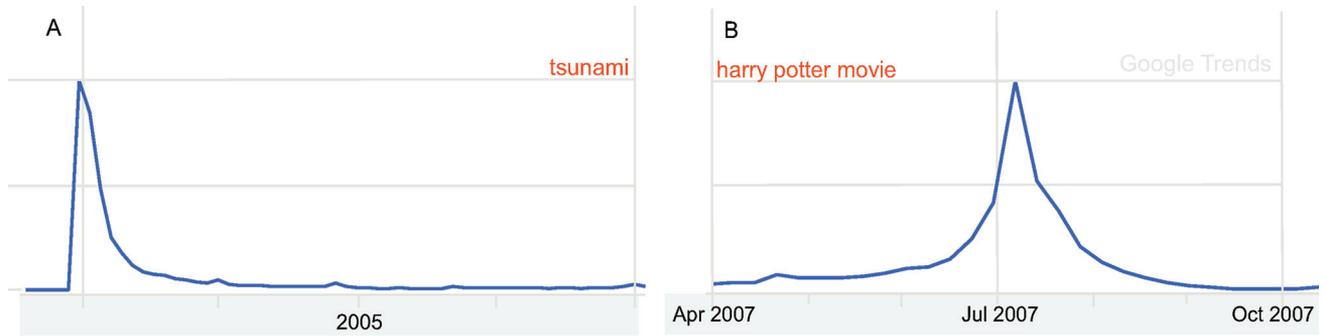


Fig. 1. Search queries as a proxy for collective human attention. (A) The volume of searches for the word “tsunami” in the aftermath of the catastrophic Asian tsunami. The sudden peak and relatively rapid relaxation illustrates the typical signature of an “exogenous” burst of activity. (B) The volume of search queries for “Harry Potter movie.” The significant growth preceding the release of the film and symmetric relaxation is characteristic of an “endogenous” burst of activity.

Such an epidemic process can be conveniently modeled by the so-called self-excited Hawkes conditional Poisson process (13). This gives the instantaneous rate of views $\lambda(t)$ as

$$\lambda(t) = V(t) + \sum_{i:t_i \leq t} \mu_i \phi(t - t_i) \quad [2]$$

where μ_i is the number of potential viewers who will be influenced directly over all future times after t_i by person i who viewed a video at time t_i . Thus, the existence of well connected individuals can be accounted for with large values of μ_i . Lastly, $V(t)$ is the exogenous source, which captures all spontaneous views that are not triggered by epidemic effects on the network.

Predictions of the Model: Dynamic Classes

According to our model, the aggregated dynamics can be classified by a combination of the type of disturbance (endo/exo) and the ability of individuals to influence others to action (critical/subcritical), all of which is linked by a common value of θ . The following classification of behaviors emerges from the interplay of the bare long-memory kernel $\phi(t)$ given by Eq. 1 and the epidemic influences across the network modeled by the Hawkes process Eq. 2.

- **Exogenous Subcritical.** When the network is not “ripe” (that is, when connectivity and spreading propensity are relatively small), corresponding to the case when the mean value $\langle \mu_i \rangle < 1$, then the activity generated by an exogenous event at time t_c does not cascade beyond the first few generations, and the activity is proportional to the direct (or “bare”) memory function $\phi(t - t_c)$:

$$A_{\text{bare}}(t) \approx \frac{1}{(t - t_c)^{1+\theta}} \quad [3]$$

- **Exogenous Critical.** If instead, the network is ripe for a particular video, i.e., $\langle \mu_i \rangle$ is close to 1, then the bare response is renormalized as the spreading is propagated through many generations of viewers influencing viewers influencing viewers and so on, and the theory predicts the activity to be described by ref. 7:

$$A_{\text{ex-c}}(t) \approx \frac{1}{(t - t_c)^{1-\theta}} \quad [4]$$

- **Endogenous Critical.** If in addition to being ripe, the burst of activity is not the result of an exogenous event, but is instead fueled by endogenous (word-of-mouth) growth, the bare response is renormalized giving the following time dependence for the view count before and after the peak of activity (7):

$$A_{\text{en-c}}(t) \approx \frac{1}{|t - t_c|^{1-2\theta}} \quad [5]$$

- **Endogenous Subcritical.** Here, the response is largely driven by fluctuations and not bursts of activity. We expect that many time series in this class will obey a simple stochastic process.

$$A_{\text{en-sc}}(t) \approx \eta(t), \quad [6]$$

where $\eta(t)$ is a noise process.

The dynamics described by the above classifications are illustrated in Fig. 2.

Predictions of the Model: Peak Fraction

In addition to these dynamic classes, the model predicts, by construction, a relationship between the fraction of views observed on the peak day compared with the total cumulative views (Fig. 2 *Inset*), henceforth referred to as “peak fraction” or F . This simple observation turns out to be a useful metric for grouping time series into categories based on whether they are endogenous or exogenous. For the exogenous subcritical class, the absence of precursory growth and fast relaxation following a peak imply that close to 100% of the views are contained in the peak. For the exogenous critical class, the fractional views in the peak should be smaller than the previous case on account of the content penetrating the network resulting in a slower relaxation. Finally, for the endogenous critical class, significant precursory growth followed by a slow decay imply that the fractional weight of this peak is very small compared with the total view count.

Results

We find that most videos’ dynamics ($\approx 90\%$) either do not experience much activity or can be described statistically as a Poisson process (verified using a Chi-Squared test). This large set of data is consistent with the endo-subcritical classification. For the remaining 10% ($\approx 500,000$ videos), we find nontrivial herding behavior that accurately obeys the three power-law relations described above. Characteristic examples of endogenous and exogenous dynamics are shown in Fig. 3.

For these videos that experience bursts, we extract the relaxation exponents using a least-squares fit on the logarithm of the data over a window of 10 days after the peak. This procedure is repeated for all possible window sizes ranging from 10 to 224 days. The “best” window is then chosen to be the largest number of days over which one can assume that the residuals of the percent deviation from the best fit are normally distributed. Once this assumption is violated at the 1% level, which occurs when the dynamics are no longer governed by Eqs. 3, 4, or 5, the fit is stopped. Although estimation of scaling laws is a subtle and controversial issue (14, 15), this procedure has been extensively tested by using synthetic data and is able to sufficiently recover exponents with an accuracy of ± 0.1 .

