THE RELATIONSHIP OF THE COMMON CORE MATHEMATICS STANDARDS TO THE VERMONT MATHEMATICS INITIATIVE PROFESSIONAL DEVELOPMENT

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Abstract

The Common Core State Standards for Mathematics (CCSSM) will be implemented nationwide as the United States strives to help students attain a higher level of mathematics achievement. Past reforms based on student learning have failed to improve student's mathematics achievement. This research, rather than focusing on student-based reforms, focuses on reform of elementary teacher's mathematics knowledge through pre-teacher training and professional development. Literature indicates that current elementary teacher preparation is inadequate and elementary teachers do not have a sufficient understanding of mathematics content. A crosswalk between one successful professional development program (the Vermont Mathematics Initiative) and the CCSSM indicates that appropriate teacher preparation with a central focus on mathematics content knowledge for elementary teachers results in positive student performance. Programs like the Vermont Mathematics Initiative (VMI) with a strong focus on content knowledge and mathematical confidence enable teachers to teach mathematics effectively so that students are better prepared to meet the CCSSM expectations.

Keywords: Common Core, mathematics, professional development, teacher knowledge.

Author's Note

The inspiration for this thesis developed as I progressed through both an elementary education and mathematics undergraduate major. People did not see how my two majors were related. Whenever I was asked what I was studying in school, they would ask if I was going to teach high school mathematics or jokingly ask if I was planning to teach calculus to elementary students. At first I was never able to come up with a simple response as to why I was working towards a degree in both mathematics and elementary education, so I just laughed and let their remarks pass. The point is that it is rare for someone to combine the mathematics route with the elementary education route. The mathematics route is focused primarily on content, which then leads to teaching high school students or university level mathematics. The other route begins with an elementary education program, which leads directly to teaching at the elementary level. As I progressed through both majors, I realized the reality is that effective elementary education teachers need the mathematics content knowledge that I received from the College of Engineering and Mathematical Sciences. The significance of this mathematics knowledge gap became openly apparent to me as I sat in education courses in which my peers in elementary education had very little knowledge of mathematics content. The 'mathematics' in the courses required of elementary education students was truly 'elementary.' After taking one of the two required elementary mathematics courses I asked my advisor if there was a way to avoid having to take the second course because I had expected to be learning how to teach elementary mathematics, not to be instructed in learning actual mathematics content at the level I would be teaching to my own elementary students. The idea that we are putting elementary teachers into schools with such weak mathematics content knowledge befuddles me. The outcome of my research in this thesis supports my belief that it is critical for elementary teachers to have a strong content understanding of mathematics, well beyond the elementary level, so they can effectively teach mathematics to their students.

Introduction

Teachers have an incredible impact on students and their success in mathematics. A child's formative years occur during elementary school and mathematics is a sequential subject. Thus, elementary teachers need to know and appreciate mathematics with the goal that their students will grow to know and love the subject. The transfer of mathematics from teacher to student plays a pivotal role in the ultimate goal of sustained mathematics learning over time. Of course there is an ever changing curriculum teachers must follow and learning objectives to be met, but the heart of learning is teaching children to find mathematics meaningful and enjoyable. Over the years various changes have been made in mathematics curriculum expectations for students, in hopes that they would be the 'silver bullet' for education reform. Yet, none of the past reforms, all focused on student learning, have positively affected student's mathematics learning.

History

Many attempts at mathematics curriculum reform have been made over the past fifty years. A brief review of these reforms follows, beginning with the latter half of the twentieth century and continuing through to present day. The reforms thus far have focused on curriculum for the students, but there is no evidence indicating that student learning has improved.

A catalyst for significant attention to be directed at mathematics was the Soviet Union's launch of Sputnik 1, in 1957, which sparked the space race between the United States and the Soviet Union (Burris, 2005). Fearful of falling behind the Soviet Union in mathematics and science, the United States government placed a priority emphasis on science and mathematics. The goal was to ensure that all students were proficient in mathematics. This emphasis on mathematics and science was a stimulus for mathematics reform.

In the 1960's, 'New Math' was implemented in classrooms across the United States. This reform consisted of content new to elementary teachers that had an emphasis on proof, abstraction, specific mathematics language, and mathematical properties. New Math failed to increase students' mathematics skills and many parents and educators found that the New Math created more confusion (Burris, 2005). Critics of New Math noted that many teachers were unprepared to teach this new content to their students, (Schoenfeld, 2005) which raised the question, "If teachers had more content knowledge would their students be more successful?" Research studies indicate yes, teachers with greater mathematics content knowledge are more capable of effectively teaching mathematics to their students. (Rivkin, Hanushek, & Kain, 2005; Boyd et al., 2012; Hill, Rowan, & Ball, 2005). However, educational reforms continue to emphasize mathematics curriculum for students rather than teacher knowledge.

A backlash against New Math arose in the late 1970's and the focus of the mathematics curriculum in schools went back to the 'basics'. Through the early 1980's there was a strong emphasis on arithmetic computation and rote memorization of algorithms and basic arithmetic facts (Burris, 2005).

Highlighting many indicators that these mathematics reforms had not improved student learning, in 1983 the National Commission on Excellence in Education presented a report entitled "A Nation at Risk: The Imperative for Educational Reform" to the Secretary of Education and the American people (National Commission of Excellence in Education, 1983). The Commission reported, "Many 17-year-olds do not possess the 'higher order' intellectual skills we should expect of them…only one-third can solve a mathematics problem requiring several steps. " and "Between 1975 and 1980, remedial mathematics courses in public 4-year colleges increased by 72 percent and now constitute one-quarter of all mathematics courses taught in those institutions." (National Commission of Excellence in Education, 1983, p. 11).

Another reform followed in 1989 when the National Council of Teachers Mathematics (NCTM) released their new curriculum and evaluation standards called the NCTM standards. This reform could be considered a backlash to the 'back to basics.' The standards stressed problem solving, communication, connections, reasoning, and critical thinking (Burris, 2005). Created by educators not mathematicians, the NCTM standards downplayed the importance of mathematics skills and content. "Teachers who had themselves been taught in traditional ways were now being asked to teach in new ways, and not given much support in doing it." (Schoenfeld, 2005, p. 21). Controversy surrounded the NCTM standards. Many distinguished mathematicians, parents, and scientists felt that the NCTM standards' focus on mathematics activities rather than mathematics skills was a very serious omission (Klein, 2003). In reaction to the standards, an adversary group called 'Mathematically Correct' was formed to oppose the NCTM standards. The debate that followed was known as the 'Math Wars' (Schoenfeld, 2005).

Revisions of the NCTM standards followed over the next few years as a result of the Math Wars and educational research calling for pedagogy reform. Studies were being presented in the 1990's, indicating that teachers should use technology and manipulatives to help students learn. To support the NCTM standards, the *Professional Standards for Teaching Mathematics* were published in 1991, describing six pedagogical standards for teachers to practice in order to support student learning in the classroom. Additionally, assessment standards for teaching mathematics were introduced four years later in 1995. The original 1989 NCTM standards were updated in 2000 in the *Principles and Standards for School Mathematics* (Burris, 2005). Focusing on what students should know, specifically in terms of curriculum was the common focus of all of the above reforms.

In 2002 when the United States Congress enacted legislation titled the 'No Child Left Behind Act' (NCLB), stringent mathematics accountability standards were imposed on schools. The government required that all states be sure schools had accountability systems (Burris, 2005). As described in a NCLB Action Brief, "The NCLB state accountability system is based on the development of state content and academic achievement standards which are measured by state assessments and compared to the 'adequate yearly progress' expectations." (NCLB Action Brief, n.d. p. 1). Not unlike earlier reforms, NCLB is supportive of standards-based education reform. Results from the National Assessment Educational Progress (NAEP) assessments indicated that there was little to no improvement in students' mathematics scores since NCLB had been passed[1] (Fair Test, n.d.).

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[1] More detailed information from "NAEP Results Show Children Still Left Behind Under NCLB" Retrieved from

The most recent reform that will be implemented in schools this year is the Common Core State Standards Initiative (CC). Following the model of previous reforms, the CC is a curriculum-based reform that focuses on what content students need to learn.

The Common Core State Standards for Mathematics

The Common Core State Standards[2] are a set of standards for English Language Arts and Mathematics that have been adopted and will be fully implemented in all but five of the fifty states[3]. The new Standards are "evidence-based[4]" with the goal of preparing students to be successful in college and to be productive members of the national workforce. The Common Core State Standards for Mathematics (CCSSM) is a part of the Common Core State Standards that detail a set of new mathematics expectations for school age children. The CCSSM consist of eleven different content areas that collectively span grades K through 12. These mathematics standards were created with the intention "to ensure that all students, no matter where they live, are prepared for success in post secondary education and the workforce." (NGA & CCSSO, 2009). A broad based working group representing multiple stakeholders, including school administrators, teachers, mathematics educators, mathematicians, statisticians, and other professionals including experts in special education and bilingual education was assembled to draft the CCSSM. The National Governor's Association (NGA)[5] and the Council of Chief State School Officers (CCSSO)[6] released the Standards for public comment in March of 2010,

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http://www.fairtest.org/sites/default/files/NAEP%20results%20show%20NCLB%20failing%202.pdf

[2] Hereafter referred to as the "Standards."

[3] As of April, 2013 states that have chosen not to adopt the Standards include Alaska, Nebraska, Texas and Virginia. Minnesota has only adopted the Common Core State Standards for English language arts. (NGA & CCSSO, 2010).

[4] "The CCSS are based on a large body of evidence, including scholarly research, surveys on the skills required to enter college and workforce training programs, assessment data identifying college- and career-ready performance, and comparisons to standards from high-performing states and nations." (National Education Association, n.d., p. 23).

[5] The NGA is a "bipartisan organization of the nation's governors" that "promotes visionary state leadership, shares best practices and speaks with a collective voice on national policy." by "identify(ing) priority issues and deal(ing) with matters of public policy and governance at the state and national levels." (National Governors Association, 2011)

[6] The CCSSO is "a nonpartisan, nationwide, nonprofit organization of public officials who head departments of elementary and secondary education in the states, the District of Columbia, the

followed by the final version of the Standards in June of 2010. Aiming to provide national consistency, the NGA and CCSSO claim to have generated a "greater opportunity to share experiences and best practices within and across states that will improve our ability to best serve the needs of students." (NGA & CCSSO, 2010). Although the standards are to be implemented by individual states, these standards set high national expectations for all K-12 students.

In summary, over the past fifty years, mathematics education has had a focus on student learning objectives that show a paucity of student improvement. Looking forward, the CCSSM is a comprehensive program of standards created by both educators and mathematicians. A primary goal of this paper is to demonstrate that a historical shift in focus from the student's curriculum to the teachers' mathematical needs is imperative in order to create positive change in student mathematics achievement.

Teacher Preparation

One way to begin to prepare teachers for the CCSSM is to ask the question of "What is different about the CCSSM?" There are six major shifts in mathematics from previous expectations to the new CCSSM. Focus, coherence, fluency, deep understanding, application, and dual intensity are the shifts highlighted for teachers (NYSED, 2012).

The first noted change in the new Standards is that the content focuses deeply on a narrower spectrum of concepts. Critics of the CCSSM claim that they leave out some important topics. On the other hand, supporters of the Standards state that they have a more concentrated focus on fewer topics. Teachers will be expected to connect the students' learning in order to build on their prior knowledge. New expectations of the CCSSM also include fluency such that students are able to make simple calculations both quickly and accurately. The CCSSM not only expects students to be fluent, but also expects students to understand the 'why' behind the algorithms they are computing. The last two shifts put expectations on

Department of Defense Education Activity and five US extra-state jurisdictions. The CCSSO provides leadership, advocacy, and technical assistance on major educational issues."(Council of Chief State School Offices, 2012).

students to be able to apply mathematics independently through practice, resulting in understanding (NYSED, 2012). All of these shifts aim to increase student's success in mathematics.

History dictates that simply informing teachers about the CCSSM and how they are different from prior standards will not be enough. Past reforms, even when coupled with professional standards for teachers as the NCTM standards were, do not give much reason to have confidence in the CCSSM being successful without extensive attention to the qualifications of teachers. Augmenting the focus on student needs to teacher needs could increase student success in mathematics. The following section takes a closer look at current elementary teacher preparation in terms of content knowledge and the adequacy of this preparation.

Is Mathematics Preparation of Elementary Teachers Adequate?

The next question to ask is, "Is the current mathematics preparation for elementary school teachers adequate?" As supported by many studies referenced in the following sections, the answer to this question is that teachers do not have a strong enough understanding of mathematics. Teacher's current preparation and mathematics knowledge is lacking.

Pre teacher training is not consistent in the United States from state to state. While nearly all states require teachers to be licensed, the undergraduate coursework, licensure requirements and tests vary across states.[7] Most teachers go through a four-year undergraduate elementary education program with some degree of student teaching experience. Many of these elementary programs do not require students to take any higher-level mathematics courses[8]. Some teachers have master's degrees in education while others earn an undergraduate degree not related to elementary education and then go through a program such as *Teach For America* to become certified (Virtual Education Software, n.d.).

[7] For a compilation of individual state requirements for teacher licensure, visit the College of Education at the University of Kentucky: http://education.uky.edu/AcadServ/content/50-states-certification-requirements
[8] To see an example of undergraduate teacher training coursework requirements see table in Appendix A, which compares literacy and mathematics courses that are required of elementary education majors at one state university.

The National Commission on Excellence in Education (1983) reported findings based on teacher preparation as follows: “not enough of the academically able students are being attracted to teaching…teacher preparation programs need substantial improvement…too many teachers are being drawn from the bottom quarter of graduating high school and college students. The teacher preparation curriculum is weighted heavily with courses in ‘educational methods’ at the expense of courses in subjects to be taught. A survey of 1,350 institutions training teachers indicated that 41 percent of the time of elementary school teacher candidates is spent in education courses, which reduces the amount of time available for subject matter courses.” (p. 19). In addition to the lack of adequate pre-teacher training, the National Commission on Excellence in Education reported that there is a serious shortage of mathematics teachers—a survey done in 1981 reported that 43 of 45 states surveyed had a shortage of mathematics teachers. Additionally, fifty percent of mathematics teachers who were recently employed were not qualified to teach mathematics (National Commission of Excellence in Education, 1983).

Current Problems: What Teachers Lack

The lack of consistent and adequate teacher training results in two main problems: many elementary teachers do not have a deep understanding of mathematics content and with this lack of content understanding a fear of mathematics often arises. One can conclude from the evidence that together, teacher’s lack of content understanding and lack of confidence in teaching mathematics contribute to the high percentage of students who are not reaching grade level mathematics expectations.

Content Understanding

Lack of deep content understanding underlies the problem of poor mathematics teaching. An analogy with reading[9] clarifies the necessity of teacher’s mathematical content understanding: Elementary teachers are capable of reading proficiently at a tertiary level. It would be unsuitable for a teacher to only be capable

[9] Most elementary teachers have a strong content knowledge of literacy. Thus it seems appropriate at times to relate teaching and understanding reading to teaching and understanding mathematics. In our highly technological world, mathematics is as important as reading and must be understood just as well by elementary teachers.

of reading at an elementary school level and teach elementary students how to read. Rather, undergraduate courses for teachers in training generally cover all aspects of how to teach reading strategies and different ways to approach reading instruction, after teachers in training have achieved a reading ability at the tertiary level. The same level of expectation must exist for teachers in regard to mathematics if we are to successfully teach mathematics to elementary students.

Overall, most teachers have completed high school mathematics, though whether they have understood mathematics at the high school level is another consideration. Quotations from elementary teachers provide further evidence on the lack of mathematics content knowledge:

> "I am really worried about teaching something to kids I may not know. Like long division—I can *do* it—but I don't know if I could really *teach* it because I don't know if I really *know* it or know how to *word* it." "I'm not scared that kids will ask me, you know, a *computational* questions that I cannot solve; I'm more worried about answering *conceptual* questions. Right now, my biggest fear—and I'm going to have to confront this on the third of February—is what I am going to do if they ask me some kind of question like, 'Why are there negative numbers?'" (Ball, 1990).

Research indicates that teachers need to have a strong knowledge base of mathematics to teach effectively (Ball, 1990; Shulman, 1986). For example, a study done by Hill, Rowan, & Ball (2005) looked at "whether and how teachers' mathematical knowledge for teaching contributes to gains in students' mathematics achievement" (p. 2). The results indicated, "teachers' mathematical knowledge was significantly related to student achievement gains in both first and third grades" (Hill, Rowan, & Ball, 2005, p. 2). Many other research studies have indicated that teachers who do not have a deep understanding of mathematics are much less capable of effectively teaching mathematics to their students (Rivkin, Hanushek, & Kain, 2005; Boyd et al., 2012). Most teachers are not consistently trained in the United States and many do not have the adequate mathematical content knowledge necessary to effectively teach students.

This weakness in mathematics is exposed in Liping Ma's, *Knowing and Teaching Elementary Mathematics*. Ma compares elementary school teachers from the United States and China. Ma documents that overall, Chinese elementary school teachers have a deeper understanding of mathematics and thus their students are more successful. Despite the fact that elementary school teachers in the United States have more years of formal schooling than Chinese teachers, when "considered as a whole, the knowledge of the Chinese teachers seemed clearly coherent while that of the U.S. teachers was clearly fragmented." (Ma, 1999, p. 107). Ma also writes that "Even expert teachers, experienced teachers who were mathematically confident, and teachers who actively participated in current mathematics teaching reform did not seem to have a thorough knowledge of the mathematics taught in elementary school." (Ma, 1999, p. xix). Teachers need to have a clear, coherent understanding of upper level mathematics so that concepts are interwoven for the teacher, allowing them to explain new concepts to students using multiple modalities and making connections to prior knowledge.

Because of the negative impact fragmented knowledge can have, teachers must have an in-depth understanding of mathematics so that this understanding can be transferred to their students. "Unlike factors outside of classroom teaching, teachers' knowledge might directly affect mathematics teaching and learning." (Ma, xix-xx).

Ma makes an important point that a teachers' knowledge has a very significant influence on his/her students. In the past, standards have focused on what students need to know, but it is just as important to focus on what teachers need to know. We conclude from all of these studies that elementary teachers must have a strong understanding of mathematics content.

Is Content Enough?

In addition to content, pedagogy plays an important role in education. Once teachers have the content knowledge necessary to teach effectively, they should be instructed in how to utilize their higher content knowledge in enhancing their instruction.

The CCSSM addresses the needs beyond content by listing eight *Standards for Mathematics Practice*. As stated in the CCSSM:

> The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). (NGA & CCSSO, 2012)

The existence of the Practice Standards reinforces the pedagogy that emanates from content knowledge. Though, without content knowledge, none of the practice standards can be implemented.

Teacher Confidence

Many elementary teachers admit that they do not feel confident teaching mathematics and that they are not 'good at mathematics'. Often teachers lacking a deep understanding of mathematical content lack confidence in their mathematics teaching. The lack of confidence is also associated with the phenomenon referred to as 'math anxiety.' Math anxiety can be defined as "a feeling of intense frustration or helplessness about one's ability to do math." (Math Academy Online, 2013). Many teachers who have math anxiety pass on their negative attitude towards mathematics to their students.

According to a study published in 2006 entitled *Mathematics Anxiety and Preservice Elementary Teacher' Confidence to Teach Mathematics and Science*, teachers with a lower level of confidence in teaching math had higher levels of math anxiety (Bursal and Paznokas, 2006). It has also been reported that math anxiety causes brain activity in the same area that registers physical pain (Harms, 2012).

Math anxiety is a significant problem in schools. Research shows that many children are prone to math anxiety. One of the main causes of this math anxiety seen in students can be traced to the teachers themselves. If a teacher has math anxiety or just doesn't like math then the students are likely to develop the same negative attitude towards mathematics. But, if teachers have self-confidence in themselves as teachers of mathematics, then their students will have more self-confidence as mathematics learners (Stipek, Givvin, Salmon, & MacGyvers, 2001). Unless teachers become positive mathematics role models, students will continue to avoid mathematics classes because of their mathematics anxiety (Lyons & Beilock, 2012).

Teachers are very powerful role models to their students, which is why it is so critical that teachers are confident in and are enthusiastic about mathematics. One mentor teacher with whom I taught shared with me that she always told her students how much they loved the subject they were working on and eventually, they did come to love the subject. In summary, in order to reduce this math anxiety in classrooms, schools need teachers who are competent in, excited about, and confident about mathematics.

Students Are Not Meeting Standards

The teacher's lack of confidence and content understanding in mathematics has a direct impact on students. As indicated by their scores on standardized testing, students across the country are failing to meet grade level standards.

Nationally, students are not meeting standards. According to the National Assessment of Educational Progress (NAEP), as of 2011 only forty percent of students in the United States were 'at or above proficient' in mathematics at grade four. Of this forty percent, only seven percent of these students scored at the 'advanced' level. Hence sixty percent of students in the United States scored below proficient in 2011 (NAEP, n.d.), Here in Vermont, 49% of fourth grade students scored at or above a proficient level in 2011, which is one of the three highest ranking states in the nation (NAEP, n.d.).

A worldwide comparison of student's mathematical success illustrates a broader comparison. The International Mathematics and Science Study (TIMSS) evaluates fourth and eighth grade students in the areas of mathematics and science internationally. For fourth grade mathematics achievement, the three top-scoring countries in 2007 were reported to be Hong Kong-China, Singapore, and Finland, while the United States ranked at number ten (Pearson, 2013). Also in 2007, for eighth grade achievement the three top-scoring countries were South Korea, Singapore, and Hong Kong-China, with the United States ranking at number twenty-one (Pearson, 2013). One inference is that students in the United States are not being given a solid foundation of mathematics at the elementary grades, and hence are behind in later grades when the content requires students to build on prior foundational concepts that have not been learned.

The data above supports the hypothesis that in order to generate positive progress changing the students' curriculum is not enough.

Teacher Expectations: What Level of Preparation is Adequate?

The last question that arises is "What level of mathematics preparation for the elementary teacher is adequate?" Adequate teacher preparation will allow teachers to effectively teach the mathematical content described in current standards. Effective teaching prepares students for later grades because mathematics is a sequential subject that builds on foundational concepts. As described above, results indicate that many students are not reaching proficiency. If teachers are not able to teach at a level that supports their students success then we need to focus on strengthening teacher preparation so they can translate their mathematics knowledge to students. Thus teachers need to be educated in a way that will allow them to be knowledgeable of mathematics at a tertiary level, competent to teach mathematics strategies to students, and knowledgeable of the skill progressions across grades.

Tertiary Level of Mathematics

The expectation must be that elementary teachers have achieved a tertiary level of mathematics understanding. This means that teachers need to take undergraduate mathematics courses that provide them with a deep understanding of mathematics content and the confidence to teach the elementary mathematics curriculum.

Toolbox of Strategies

When teachers have a strong understanding of content, they are able to teach their students the strategies or 'tools' they need to be successful. An analogy with reading clarifies the significance of teaching tools in mathematics. Teaching reading strategies in terms of a 'reading toolbox' is one of many effective ways to approach reading instruction in an elementary classroom. This 'reading toolbox' can be defined as a set of reading strategies for teachers, which enable teachers to be strong and confident readers. Not only do teachers use their own reading toolboxes daily to teach their students, but they also teach students to build their own reading toolboxes. When students get stuck on a word while reading, they can problem solve using tools in their toolbox to figure out the word. It is important to note that a child cannot be taught every single word that they will ever encounter. Rather, children must be taught strategies so that they will be able to figure out how to read the countless new words that they come across.

Similarly, every teacher needs to have a mathematics toolbox that consists of a set of problem solving strategies so that when stuck on a math problem multiple approaches are available to solve the problem. Again, it is important to note that children cannot be taught how to solve every single math problem that exists. Instead, students must be taught strategies so that they will be able to tackle the problems they encounter. In order for this to happen, teachers must have their own toolbox that is filled with strategies to teach to students. A teacher without a toolbox will not be able to teach their students to have toolboxes of their own. See Appendix B for introductory mathematics toolbox lesson plan, which I wrote and used in my student teaching.

Skill Progressions

Mathematics toolboxes are especially important for teachers and students as they build on concepts over time. Commonly referred to as 'progressions,' there are specific ways that concepts are built up when teaching students, often across grade levels, when teaching students. It is important that teachers have knowledge about the content in the grades above and below the grades they teach so they can understand the skill progressions from start to end. To continue with a reading analogy, consider skill progressions in both reading and mathematics. If children were taught how to read words before they knew their letter sounds, they would not have a strong foundation of knowledge to support them in being successful. The same idea follows in mathematics; if students are not effectively taught the number concepts in the elementary curriculum, they will not be able to successfully learn new content in higher grades because foundational building blocks will be missing.

There are many progressions throughout mathematics. Since the CCSSM has been released, a team from the University of Arizona has been working on progressions relating to the content in the CCSSM[10]. The significance of these progressions is to "explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give mere detail on particularly knotty areas of the mathematics." (AZ Board of Regents, 2007). It is important that students have a strong understanding of the beginning of progressions before they are taught the concepts that build on initial progressions in mathematics.

Additionally, teachers should be aware of skill progressions across grades so they know which parts of the curriculum need to be prioritized. Knowing how the students use the math at their current grade level as well as in later grades enables teachers to confidently make these curriculum decisions.

One remedy for the insufficient mathematics preparation for elementary teachers is to provide current teachers with important knowledge and skills through professional development that has a strong foundation in content knowledge. One specific example of this in Vermont is a professional development program named the Vermont Mathematics Initiative (VMI). In fact, the VMI was created in response to teachers in Vermont who recognized that they were lacking sufficient mathematics content knowledge to teach effectively.

The Vermont Mathematics Initiative

[10] These progressions are still in draft form, open for public comment. They were released through University of Arizona's Institute for Mathematics and Education.

The genesis of the Vermont Mathematics Initiative (VMI) can be traced to the years 1996-1999 when Dr. Marc Hull was Vermont's Education Commissioner. Dr. Hull conducted action planning institutes across the state to assess teacher needs. The number one need voiced by elementary teachers across the state was stronger knowledge of mathematics content. The State Department of Education then asked Dr. Kenneth I. Gross, a mathematics professor at the University of Vermont, to develop a program to do so. Thus in 1999 the VMI was founded (Rathke, 2001).

VMI is a program designed to increase the mathematics capability of practicing K-8 teachers throughout Vermont. From the start, the goal of this program was to emphasize strong mathematical content knowledge– in areas that include arithmetic, algebra, geometry, trigonometry, number theory, probability, statistics, and calculus – and the capability to translate content knowledge into effective classroom practice. The philosophy of this program can be summarized by the phrase "competence leads to confidence" and it has attracted many elementary school teachers who had significant weaknesses in mathematics.

The VMI model combines two stages, referred to as 'Phase I' and 'Phase 2.' Phase I is a three-year master's degree program consisting of 36 credits earned from twelve courses that train elementary teachers to be teacher leaders. Teachers who have completed Phase 1 then serve as mathematics resources to other teachers in their school or district. Teacher leaders provide an exceptional support system across the state to teachers in their schools, districts, and those who are enrolled in Phase I of the program. See Appendix C for a map of teacher leader distribution across the state (VMI, 2013).

Phase II is a six credit, 80-hour program focused on mathematical understanding of arithmetic, functions, and the relationship between arithmetic, algebra and geometry. Introduced in 2006, the ultimate goal of Phase II is to reach every elementary and middle school teacher in the state (VMI, 2013).

Vermont Mathematics Initiative Outcomes

Evaluation studies indicate that the VMI program has a positive effect in the classroom. When compared to students in control schools, students in VMI schools scored higher on statewide standardized testing. Students in schools with higher concentrations of VMI teachers “progressed at a rate more than three times that of their peers in either the group of schools having a single VMI teacher or the group of Control schools having no VMI teacher.” (Meyers & Harris, 2008, p. 1). The data also showed that that “the free or reduced lunch eligible students in the VMI schools significantly out-scored their free or reduced lunch eligible peers, and they gained on students who are not eligible for free or reduced lunch in the matched schools.” (Meyers & Harris, 2008, p. 2).

In addition to the significant student gains, teachers report improved confidence. After participating in one or more VMI courses teachers have shared the following comments through course evaluations:

> “I have seen myself gain confidence…There are many aspects of my own personal mathematical thinking that have changed and I plan on using my new knowledge and understanding with staff and students.” “This course has boosted my content knowledge tremendously.” “The VMI experience has helped me realize all methods of understanding math hadn’t been utilized.” “I feel very empowered as a learner…the why and how something works AND where it leads to in higher level math courses.” “My attitude has changed.” “The class has been incredibly valuable for me, as a learner. Because I was not confident in my own math abilities/knowledge, I have always felt like I am not a good math teacher. I now feel excited about the prospect of learning more and more about math so that my teaching can improve.” “I have gained confidence in my math skills while understanding where arithmetic-geometry-algebra correlate with my state standards.” “The VMI experience has had an extremely powerful impact on my enthusiasm toward math.” “I learned so much. In high school it took me 2 years to get through Algebra I and then I went to vocational school. In college, I only tested into basic math course. Took a lot of classes on number sense for my early childhood degree and a lot of stats to get my psyc degree. Then I taught K, 1 for 10 years—so this is new to me. I increased my confidence and ability to communicate math.” “I have grown in my confidence in math. I am learning to expand the way I explain math to my students.” “There were gaps in my learning and this class filled in those gaps! Its lasting benefit will

> be that I will approach teaching math more confidently." "I discovered that there are many gaps in my math education and VMI is filling in these gaps. I will allow my students to have more hands-on time and encourage them not to look for correct answer but to think the problem through." "This experience has left me feeling much more confident in my skills." "I have hope and increasing optimism about my ability with math. I have faced my childhood demon and I am conquering my fears." (VMI, 2013)

These comments illustrate the positive effect VMI has had on teachers. These responses also call attention to the need to reach the countless teachers across the country, who have not gone through a program such as VMI, who still have the insecurities in mathematics that the VMI teachers have overcome.

Crosswalk Between the CCSSM and the VMI

This section provides a review of the setup and content in the CCSSM and the VMI. The objective is to create a crosswalk that maps the CCSSM domains to the VMI topics in order to determine if the VMI professional development prepares teachers to effectively teach the content in the CCSSM to their students.

The CCSSM is organized as follows: There are eleven main content areas called domains. Within each domain there are multiple clusters of content that are contained in the given domain. More specifically, each cluster has a series of standards within it that students are expected to achieve. See Appendix D for a complete list of domains and clusters for grades K-6.

The VMI master's degree curriculum that trains teacher leaders consists of the following 12 courses:

Mathematics as a Second Language (MSL)

Functions, Algebra, & Geometry, I (FA&GI)

Functions, Algebra, & Geometry, II (FA&GII)

Probability, Measurement, and Geometry (PMG)

Number Theory (NT)

Statistics, Action Research, and Inquiry into Effective Practice, I

Statistics, Action Research, and Inquiry into Effective Practice, II

Functions, Algebra, & Geometry, III (FA&GIII)

Statistics, Action Research, and Inquiry into Effective Practice, III

Calculus for K- 8 Teachers, I

Calculus for K- 8 Teachers, II

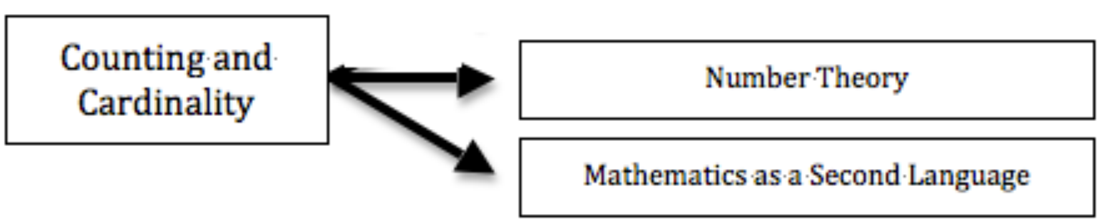
Capstone VMI Experience

Four of these courses, MSL, FA&GI, PMG, and NT are directly relevant to the CCSSM. Each course has a number of content areas called units and each unit contains a set of topics. See Appendix E for descriptions of each VMI course and Appendix F for a list of units and topics in the above four courses (VMI, 2013). The analysis of the crosswalk demonstrates that there are no content gaps between the CCSSM and the VMI.

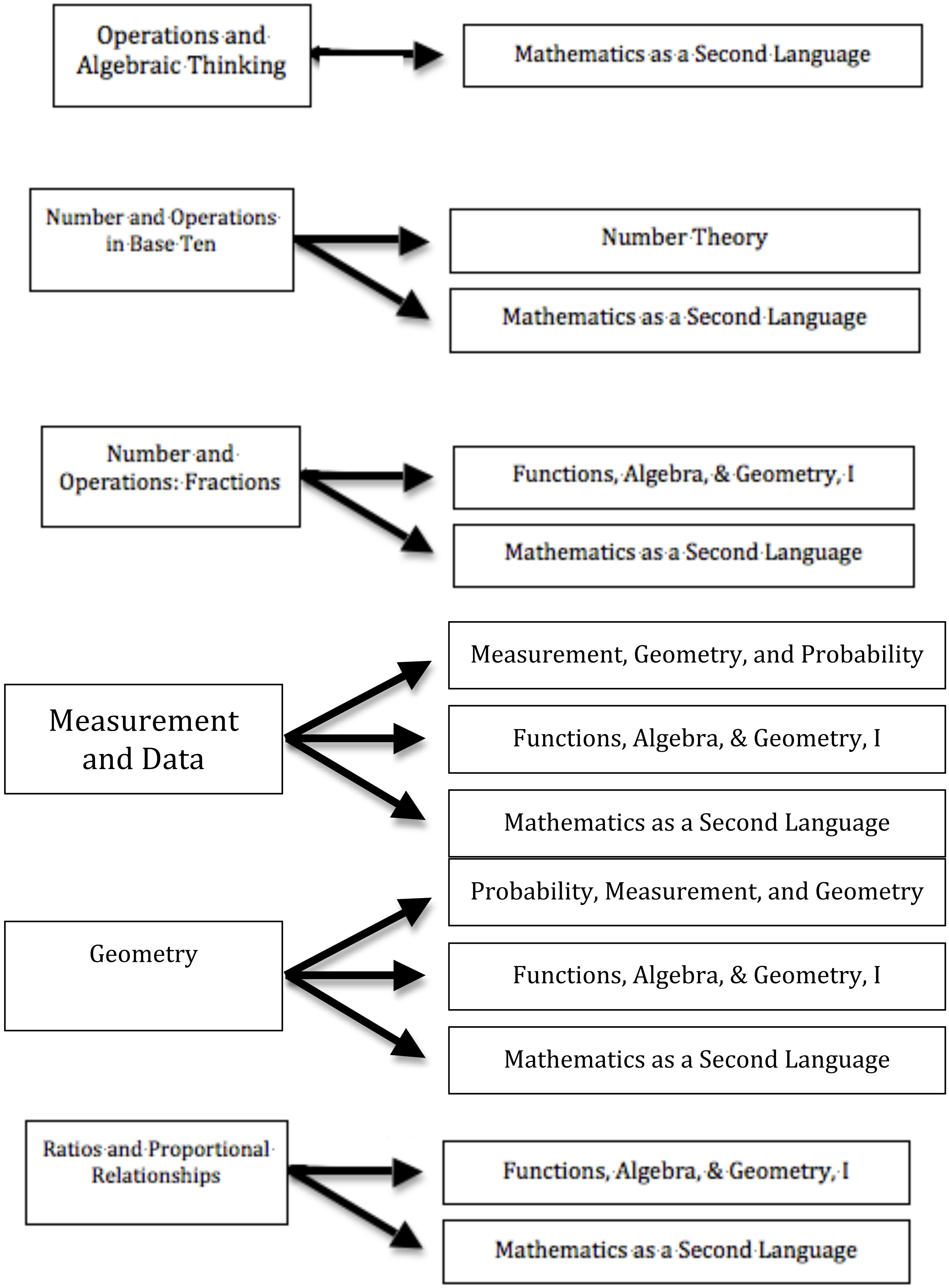
Crosswalk Comparisons

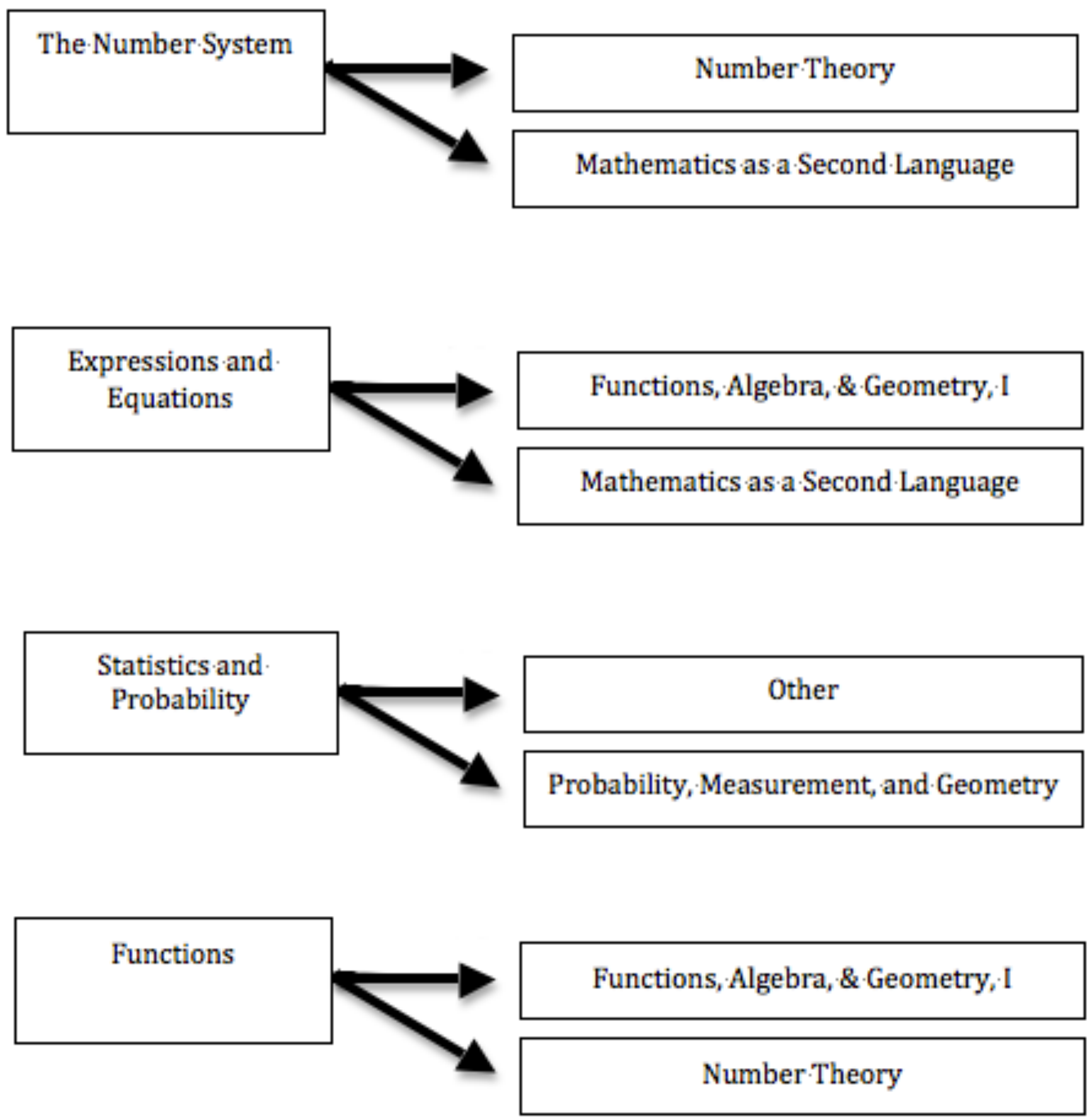
The CCSSM consist of content knowledge for students and the VMI consists of content knowledge for elementary teachers to enable them to teach the content their students need to know. The crosswalk between the CCSSM and VMI maps CCSSM domains to various VMI units that prepare the teacher to teach the given domain. The majority of the domains can be mapped to multiple topics in the four VMI courses listed above. The other eight VMI courses contain information that has a less apparent relationship to the CCSSM domains, yet is still necessary for teachers understanding of mathematics that goes beyond the level they teach.

The following diagrams illustrate the relationship between the CCSSM and the VMI.



Note: All of the standards in the domain *Counting and Cardinality* can be readily accessed by teachers who have a solid understanding of the VMI courses listed on the right side of the diagram. The selected VMI courses include all of the content that is necessary for teachers to know to effectively teach their students the content in the *Counting and Cardinality* domain. The same follows for each of the other domains.


Operations and Algebraic Thinking
Mathematics as a Second Language
Number and Operations in Base Ten
Number Theory
Mathematics as a Second Language
Number and Operations: Fractions
Functions, Algebra, & Geometry, I
Mathematics as a Second Language
Measurement and Data
Measurement, Geometry, and Probability
Functions, Algebra, & Geometry, I
Mathematics as a Second Language
Geometry
Probability, Measurement, and Geometry
Functions, Algebra, & Geometry, I
Mathematics as a Second Language
Ratios and Proportional Relationships
Functions, Algebra, & Geometry, I
Mathematics as a Second Language



Significance of Crosswalk Relationships

The topics in the VMI courses comprise important content for teachers to know because they provide teachers with two significant benefits: they gain a more thorough understanding of mathematical concepts as well as perspective on their students' future learning in mathematics. This is significant because teachers need to understand the content their students will learn at higher grade levels. For example, students need to learn about angles and triangles so that when they are exposed to topics like trigonometry they will have the foundational building blocks they need to be successful. As another example, place value and fluency with multiplication are prerequisites for learning about exponential growth in later grades. In general, all higher-level mathematics courses rely heavily on the critical foundations students learn in elementary school.

Aside: Within the group of eight VMI courses there are three courses that focus on statistics. Although not directly related to the CCSSM for the elementary grades statistics is important for teachers. Schools have become data driven, in part due to the NCLB act. Thus, teachers should be comfortable analyzing data and using it to direct their teaching. Using data to guide teaching makes for teaching that is more relevant to students needs. The statistics courses in the VMI professional development focus on understanding and developing educational research to help teachers be more effective in their teaching.

Does VMI Meet Needs of Effectively Implementing the CCSSM?

For the implementation of the CCSSM to be successful teachers should have a strong content knowledge in mathematics, confidence to teach mathematics and the pedagogy to help translate their mathematics knowledge to their students.

An analysis of teachers' pre and post scores in six of the VMI courses indicates that teachers made significant content gains throughout each course.[11] Across six VMI courses and ten years, the mean pretest score of an elementary teacher coming into the course was a 46.31 percent. The mean post-test score, after the elementary teacher had gone through the entire course was 71.50 percent. Overall, 25.19 percent increase[12] in elementary teachers scores indicates that there were significant content knowledge gains across all courses. . See Appendix G for complete data results.[13]

More specifically, VMI courses that were later in the course progression had more considerable gains from pre to post test. This could imply that teachers initially reconstruct their foundation of mathematical knowledge in earlier courses,

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[11] Data was collected through pre-tests and post-tests administered to VMI participants.
[12] 25.19 percent was the mean improvement from pre to post test. The median percentage improvement was 24.14 percent.
[13] Each pre/post test was evaluated for reliability using a Cronbach alpha coefficient. "All measures had sufficient reliability upon which to gauge teacher performance on the knowledge tests" (Meyers & Harris, 2012). See Appendix H for more information.

showing average improvements.[14] Then, they take higher-level courses and show much greater improvements[15] because they are able to use their strengthened foundation of mathematics knowledge to understand more difficult content. These data confirm that the VMI empowers teachers by providing them with a strong content knowledge to support them in effectively teaching mathematics to their students.

The crosswalk indicates that there is a strong relationship between the CCSSM and VMI. Data shows that VMI provides sufficient content knowledge and confidence to improve teachers teaching and students test results. Thus the conclusion is that VMI adequately trains teachers so they have learned all the content necessary to teach their students according to the CCSSM.

Recommendations

In order for implementation of the CCSSM to be successful, teachers need professional development that will provide them with the necessary content knowledge as the VMI program does. The history of mathematics curriculum reform in the latter half of the twentieth century has revealed that identifying 'important' mathematics content for curriculum reform does not automatically result in improved student learning, indicating that standards are not necessarily the solution for student success. Student learning depends on the teacher. Thus, the key to students being successful in mathematics is having competent and confident mathematics teachers who love to teach mathematics to their students. In order for this to be possible, undergraduate coursework and/or professional development must provide profound and meaningful mathematics instruction in both content and pedagogy for elementary teachers. The CCSSM could be a success, if elementary teachers are better prepared through intensive mathematics programs such as the VMI.

Mathematics Specialists

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[14] Elementary teachers had a mean improvement of 10.87 percent in the first VMI course *Mathematics as a Second Language*. Note this could be due to a ceiling effect.

[15] Elementary teachers had a mean improvement of 44.39 percent in Calculus, and 31.39 percent increase from pre to post test scores in the *Number Theory* course.

One way to begin the process of educating elementary teachers in mathematics is with mathematics specialists. Mathematics specialist support and train both teachers and students in mathematics.

Initially, many schools employed reading specialists who run programs like Reading Recovery[16] or work with small groups of students who need additional support in reading. Reading specialists in elementary schools also see individual students one-on-one or teach lessons with or instead of the classroom teacher. They help students expand their reading toolboxes by teaching them more reading strategies so they can be more successful and catch up to grade level expectations.

More recently, mathematics specialist and coaches have been placed in schools across the country "to construct leadership roles and to provide on-site, collaborative, professional development…to enhance instruction and to improve student achievement" (Campbell & Malkus, 2011, 431). Mathematics specialists serve the same purpose for mathematics as do reading specialists for reading. One benefit of this model is that unlike a classroom teacher who needs to learn the entire elementary school curriculum, a mathematics specialist can focus specifically on the mathematics curriculum for all grades. This not only allows more focus on the mathematical content, but also provides the mathematics specialist a greater understanding of the concepts of mathematical progressions across grade levels. Knowing what knowledge a student has, as well as what the student must learn over the next few years is invaluable. Mathematics specialists have had a "significant positive impact on student achievement." (Campbell & Malkus, 2010, 1). Mathematics specialists and coaches are reducing the number of teachers that need to be trained and positively influencing student outcomes for the entire school.

Teacher Training

Looking specifically at the VMI professional development, the four core courses of VMI contain content that adequately prepares teachers to teach the content in the CCSSM standards. It is critical that, as a bare minimum, all elementary

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[16] "Reading Recovery is a highly effective short-term intervention of one-to-one tutoring for low-achieving first graders…Individual students receive a half-hour lesson each school day for 12 to 20 weeks with a specially trained Reading Recovery teacher." (Reading Recovery Council of North America, 2012).

teachers have some kind of undergraduate training that is comparable to these courses so as to ensure that all elementary teachers have a minimal mathematics background. Training focused on teachers content and pedagogical needs rather than student's needs is essential.

In addition to this core curriculum, the content of the other eight VMI courses should be required as professional development once teachers are employed in schools. In this way, teachers would have students to work with so they could implement new learning. Teachers would be able to bring student work into their VMI courses and be able to practice and apply what they learned from their VMI coursework in the classroom. These eight courses encourage continued mathematics learning and strengthen teachers understanding of mathematics on a larger scale—knowing where their students start and where they will end up after years of mathematics.

Conclusion

Student learning depends on the teacher. Teachers and administrators must understand what training is necessary to support teachers in implementing the CCSSM. Currently, most teachers are not prepared to teach mathematics effectively to their students, and the CCSSM will boldly illustrate this deficit if professional development is not prioritized. Elementary teachers need to have a deep understanding of mathematical content knowledge and the importance of both problem solving skills and rote memorization, while still understanding the why behind the algorithm. Programs such as the VMI provide effective professional development to support teachers in increasing their content knowledge and applying it in the classroom. Analysis of the CCSSM and the VMI supports content driven professional development and recommends strengthening undergraduate teacher education programs.

The ultimate goal is to benefit students, who deserve to have teachers who are mathematically knowledgeable. Students are more likely to become proficient in mathematics and more likely to achieve success throughout later grades, in college, and in their careers if their teachers have a strong understanding of mathematics.

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Appendix A

A comparison of literacy and mathematics requirements in an undergraduate program for elementary education majors.

General Education Courses	Math 15: Elementary School Math	ENGS 001
	Math 16: Fundamental Concepts of Elementary School Math	OR English literature
Education Specific Courses	ELED 156 Teaching Math for Meaning	EDEL 175 Lab Experience in Literacy
		EDEL 176 Language Arts and Literacy Skills
		EDEL 177 Children's Lit and Literacy
		EDEL 187 Plan, Adapt, Delivery Reading Instruction

Appendix B

Mathematics Toolbox Lesson Plan

Written and used by Amanda Auger during student teaching.

Vital Information
Subject: Mathematics
Topic or Unit of Study: Mathematical Strategies (this lesson can be used with any unit in mathematics)
Grade/Level: Elementary School
Teacher Goals: Support each and every student in filling their toolbox with strategies they understand and will be able to use independently during mathematics.

Implementation
Learning Context: Students have been working on (insert given topic here). They have learned multiple approaches to solve the problems and now will summarize this knowledge by making toolboxes and putting 'mathematical tools' in them.

Student Learning Objectives: Students will be able to make sense of problems and persevere in solving them. Students will be able to reason abstractly and quantitatively. Students will be able to construct viable arguments and critique the reasoning of others. Students will be able to model with mathematics. Students will be able to use appropriate tools strategically. Students will attend to precision. (NGA & CCSSO, 2012).

Assessment of Stated Objectives: Students will be given a problem set to solve independently. They will be able to use their toolboxes during the assessment. Each question will have two parts. The first part will have the student solve a problem. The second part will ask the student what tool they used from their toolbox. Students will be asked to explain why they picked the tool, how they used it, and if there are any other tools that would help them get the same answer.

Differentiation: Any student who receives support in writing (specifically for younger grades) will have an adult to scribe for them. Students will receive one of three problem sets, depending on their readiness with the concepts. The main goal of this lesson is that students will be able to independently use mathematical strategies so the difficulty level of the problems will range based on students independent levels. As always in mathematics there should be challenge problems for any student to work on if they finish the assignment before their peers. These challenge problems should be thought provoking and challenge students to think outside of the box. Students with significant needs will be accommodated as required by their IEP.

Procedures: Start the lesson by asking students to share some of the strategies they have been using to solve mathematics problems. Write all ideas down on a big sheet

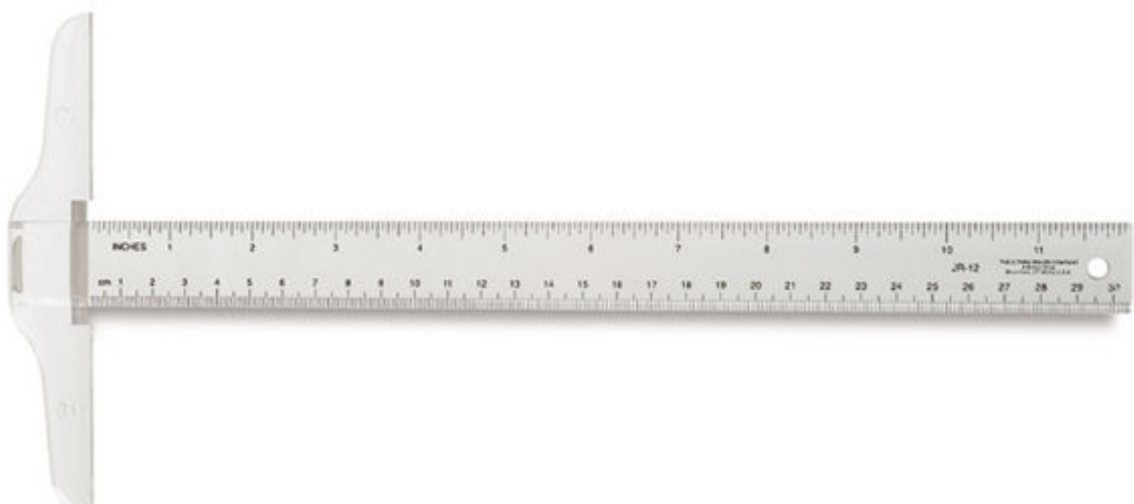
of poster board. Then explain to students that great builders don't just use one tool to build strong buildings. They need nails to hold the wood together, and a hammer to put the nails in etc. Tell students that mathematicians are just like builders. They don't just use one or two strategies when they are solving mathematics problems. They use an entire toolbox of strategies. Then inform students that they will create their own math toolboxes. Show sample toolbox and explain why each tool was put with each strategy. Have three students give examples of tools they will pick and what strategy will go with them. Have students go back to their seats and pass out supplies. As students are working be sure to have conversations with them about the strategies they are selecting and why they assigned them to a specific tool to reinforce the meaning behind the lesson. Note: for the following lessons have students use their toolboxes during math lessons.
Part 2: Assessment. Give students math problems to solve individually. Have them use their toolbox and be sure the problems given to each student are at an independent level. The important part of the assessment is to be sure students understand why they chose the strategy they did, and if there are other strategies they could also use.

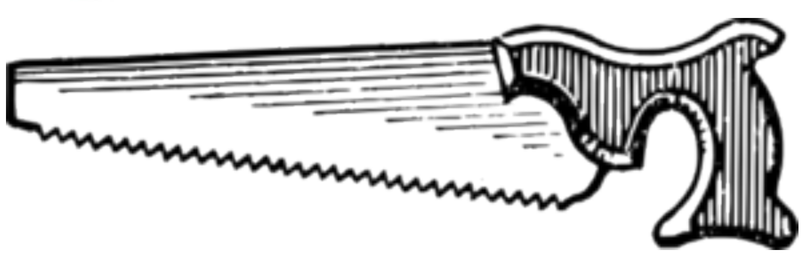
Collaboration: Students will work at tables to create their toolboxes. They will share supplies and be able to talk about what different strategies should be assigned to the various tools. Students who have finished will then be assigned a small group to discuss their favorite strategies and collaborate to decide on which few they feel are most useful for the unit they are currently studying.

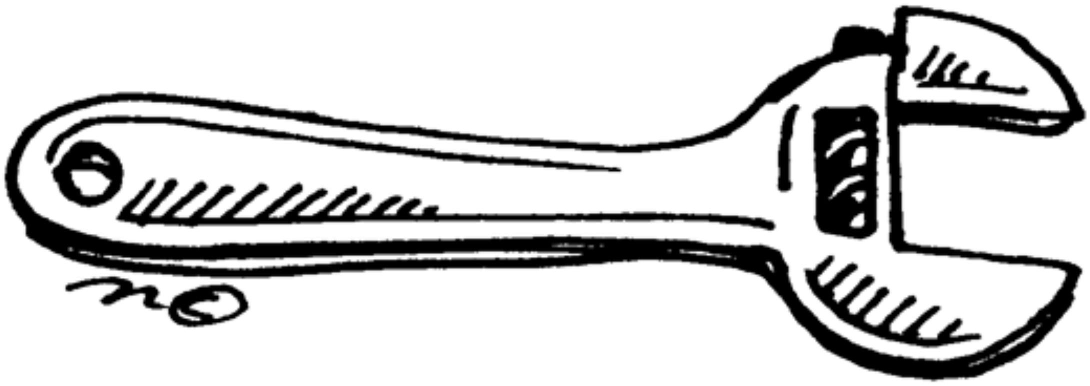
Resources and Materials: Construction paper, pipe cleaners, scissors, glue, 'tools' worksheet, single hole punch crayons or markers and pencils.

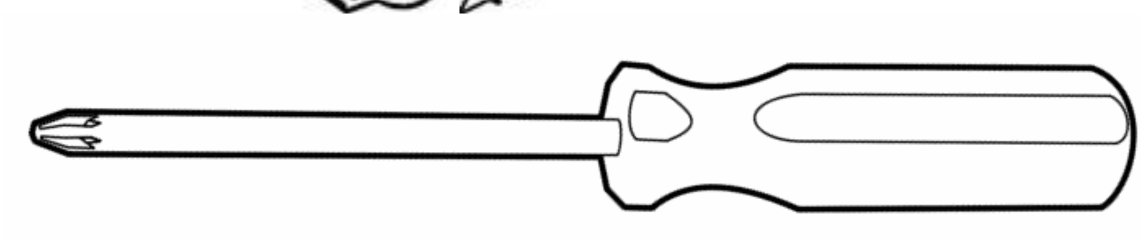
Time Allotment: Introduction and toolbox creating: 45 minutes
Assessment using toolbox: 30 minutes

Sample reading toolbox:



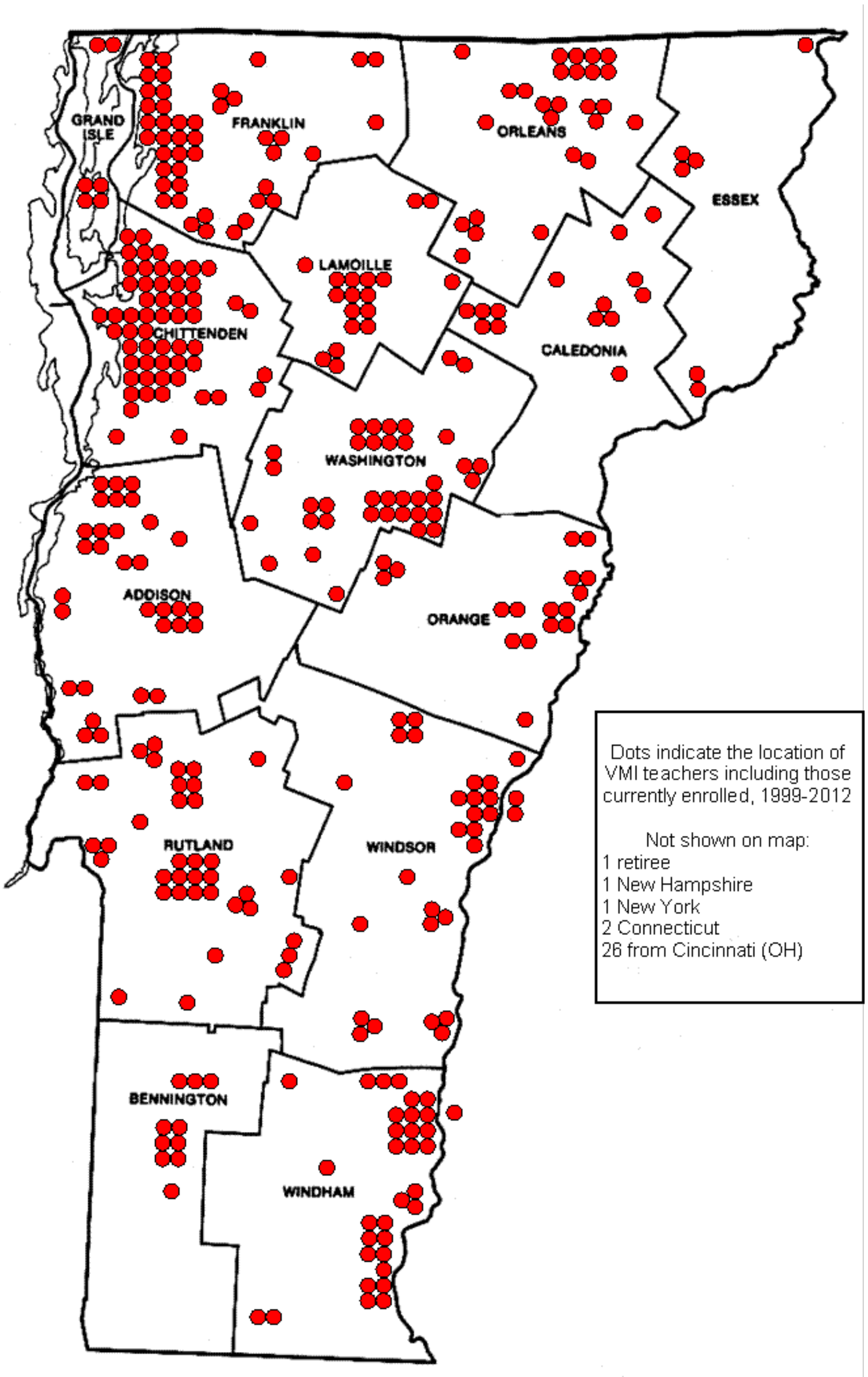






Appendix C

A map of VMI teacher leader distribution across the state of Vermont. VMI teacher leaders have completed Phase I of the VMI program.



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Appendix D

A list of the CCSSM domains (in bold) and the clusters that fall within each domain, for grades K-6.

Counting and Cardinality
Know number names and the count sequence.
Count to tell the number of objects.
Compare numbers.
Operations and Algebraic Thinking
Understand addition, and understand subtraction.
Represent and solve problems involving addition and subtraction.
Understand and apply properties of operations and the relationship between addition and subtraction.
Add and subtract within 20.
Work with addition and subtraction equations.
Work with equal groups of objects to gain foundations for multiplication.
Represent and solve problems involving multiplication and division.
Understand properties of multiplication and the relationship between multiplication and division.
Multiply and divide within 100.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Use the four operations with whole numbers to solve problems.
Gain familiarity with factors and multiples.
Generate and analyze patterns.
Write and interpret numerical expressions.
Analyze patterns and relationships.
Number and Operations in Base 10
Work with numbers 11-19 to gain foundations for place value.
Extend the counting sequence.
Understand place value.
Use place value understanding and properties of operations to add and subtract.
Use place value understanding and properties of operations to perform multi-digit arithmetic.
Generalize place value understanding for multi-digit whole numbers.
Understand the place value system.

Perform operations with multi-digit whole numbers and with decimals to hundredths.
Number and Operations-Fractions
Develop understanding of fractions as numbers.
Extend understanding of fraction equivalence and ordering.
Build fractions from unit fractions
Understand decimal notation for fractions, and compare decimal fractions.
Use equivalent fractions as a strategy to add and subtract fractions.
Apply and extend previous understandings of multiplication and division.
Measurement and Data
Describe and compare measurable attributes.
Classify objects and count the number of objects in each category.
Measure lengths indirectly and by iterating length units.
Tell and write time.
Represent and interpret data.
Measure and estimate lengths in standard units.
Relate addition and subtraction to length.

Work with time and money.
Solve problems involving measurement and estimation.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
Geometric measurement: recognize perimeter.
Solve problems involving measurement and conversion of measurements.
Geometric measurement: understand concepts of angle and measure angles.
Convert like measurement units within a given measurement system.
Geometric measurement: understand concepts of volume.

Geometry

Identify and describe shapes.
Analyze, compare, create, and compose shapes.
Reason with shapes and their attributes.
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
Graph points on the coordinate plane to solve real-world and mathematical problems.
Classify two-dimensional figures into categories based on their properties.
Solve real-world and mathematical problems involving area, surface area, and volume.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

Apply and extend previous understandings of multiplication and division.
Compute fluently with multi-digit numbers and find common factors and multiples.
Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.
Reason about and solve one-variable equations and inequalities.
Represent and analyze quantitative relationships between dependent and independent variables.

Functions (starts in 8th grade)

Statistics

Develop understanding of statistical variability.
Summarize and describe distributions.

Appendix E

Note: Course descriptions are reprinted with permission from the Vermont Mathematics Initiative.

The Vermont Mathematics Initiative Master's Degree Curriculum

The 12 VMI courses below comprise the 36-credit Vermont Mathematics Initiative Master's Degree curriculum.

Course 1: *Mathematics as a Second Language*

This course lays the groundwork for all the Vermont Mathematics Initiative courses that follow. A major theme is understanding algebra and arithmetic through language. The objective is to provide a solid conceptual understanding of the operations of arithmetic, as well as the interrelationships among arithmetic, algebra, and geometry. Topics include arithmetic vs. algebra; solving equations; place value and the history of counting; inverse processes; the geometry of multiplication; the many faces of division; rational vs. irrational numbers; and the one-dimensional geometry of real numbers. In K-12 application of content, teachers will examine the Common Core State Standards and demonstrate an understanding of how the above concepts develop across the grades.

Course 2: *Functions, Algebra, and Geometry I*

This course builds upon the prior course *Mathematics as a Second Language* and extends and reinforces the learning from that course. Participants will obtain deep understanding of the concept of a function, utilize functions in problem solving, appreciate the pervasiveness of the function idea in the K-8 mathematics curriculum as well as everyday life, and engage in a variety of problem-solving activities that relate directly to the K-8 mathematics classroom. Topics include functions, graphs, inverse functions, linear functions, the algebra and geometry of straight lines, solving linear equations and inequalities, and an introduction to nonlinear functions. This course together with *Mathematics as a Second Language* serve as the mathematical foundation for a K-8 lesson study project that participants undertake during the school year.

Course 3: *Functions, Algebra and Geometry II*

This course continues the study of algebra from the perspective of K-8 mathematics. The first part of the course is devoted to quadratic functions, parabolas, and related problem solving. In the second part the focus is on complex numbers with applications to geometry in two dimensions. The third part is centered on exponents and includes exponential functions with different bases, compound interest, and problem solving applications. Also, teachers will develop effective content-based questioning techniques and explore the components of building successful mathematics lessons.

Course 4: *Measurement and Probability*

This course builds on prior courses in algebra and geometry. Topics include measurement (length, area and volume), experimental and theoretical probability, and the ways in which these concepts develop across the elementary, middle and high school curricula. Topics are presented in the context of problem solving, and there is an emphasis on reinforcing one's understanding of functions, function notation, and topics from algebra.

Course 5: *Number Theory*

This course introduces teachers to the branch of mathematics known as number theory. Emphasis in this course is placed on the mathematical content of number theory and on how number theory is taught in grades K-8, with particular attention to student learning of number theory in these grades. Topics include the division algorithm, properties of prime and composite numbers, the sieve of Eratosthenes as a way of understanding distributions of primes and composites, the infinitude of primes, the fundamental theorem of arithmetic, properties of the greatest common factor and methods of computing the greatest common factor including the Euclidean algorithm, properties of least common multiples, use of base ten and expanded notation, writing numbers and computing in different bases, and arithmetic progressions.

Course 6: *Statistics, Action Research, and Inquiry into Effective Practice, I*

This course provides an introduction to statistics and begins to incorporate research in mathematics education. Topics include graphical and numerical organization and presentation of data, summary statistics for quantitative data, measures of relationship between variables, and inference from sample data to populations. This course forms the foundation for later work in statistics and school-based research, and is followed by the completion of a small-scale classroom inquiry. The inquiry allows participants to bring together the research they read with the statistics they learn to formulate the study, develop an intervention, and analyze the resulting data.

Course 7: *Statistics, Action Research, and Inquiry into Effective Practice, II*

This course is designed to build upon previously completed introductory work in statistics. Teachers will apply their understanding of statistics to interpret and critique educational research studies, to develop and analyze the effectiveness of classroom interventions and to analyze and interpret local assessment results. This course will prepare teachers to lead their schools in understanding the meaning and appropriate uses of assessment data and making data-driven classroom decisions. Statistics topics include measures of central tendency and variability; representations of data; probability distributions; normal curve, stanine; estimation-standard error, margin of error, confidence intervals; and hypothesis tests.

Course 8: ***Functions, Algebra and Geometry, III***

This course builds on the arithmetic, algebra, and geometry developed in prior courses. The first part of the course develops the subject of trigonometry from the perspective of the K-8 mathematics classroom. Topics include similar triangles, the trigonometric functions and their graphs, periodic functions, the number pi, and applications to measurement, wave motion, and problem solving. The second part of the course is a continuation of the study of exponential functions, and includes the number *e*, logarithm functions; applications to growth and decay; applications of logarithms in everyday life; and the history of exponential functions and logarithms. Participants also study current research on mathematics education and analyze the mathematics content and teaching skills necessary to help students develop additive, multiplicative, and proportional reasoning.

Course 9: ***Statistics, Action Research, and Inquiry into Effective Practice, III***

This course builds on prior courses in statistics and action research. The course reviews earlier concepts in descriptive and inferential statistics, and includes additional topics in the analysis of cross-tabulated data and in the analysis of correlational relationships between dependent and independent variables. Teachers will do critical reading of research on instructional practices in elementary mathematics, and will complete the design of their own action research investigations.

Course 10: ***Calculus for K-8 Teachers, I***

This course builds upon prior courses in arithmetic, algebra, and geometry. It is designed to introduce teachers to the branch of mathematics known as calculus in a way that relates calculus to the mathematics taught in the K-8 classroom. Topics include the idea of a limit, the role limits play in K-8 mathematics, the concept of instantaneous change, the derivative of a function, and applications to optimization. Course goals include reinforcing and extending arithmetic, algebra, and geometry knowledge and skills through problem solving involving calculus, and empowering teachers with a deep understanding of how capability in K-8 arithmetic and algebra is foundational for success in higher-level mathematics. This course also includes an analysis of the Vermont Grade Expectations and of the various curricula used in Vermont's schools to identify the underlying role or appearance of ideas from calculus. Participants will discuss ways to build such foundational skills and concepts into K-8 lessons.

Course 11: ***Calculus for K-8 Teachers II***

This course continues the study of calculus and its relationship to the K-8 classroom. Topics include infinite series, calculation of area, the definite integral, and the Fundamental Theorem of Calculus – all viewed from the perspective of the K-8 classroom teacher. This second course in Calculus prepares the participant to develop a 'Calculus in the Classroom' lesson to be taught during the academic school year.

Course 12: ***Capstone VMI experience***

The Capstone is the final course of the Vermont Mathematics Initiative and provides opportunities for participants to synthesize the coursework, field experiences, leadership, and research components of the VMI. Teachers will revisit key mathematical concepts from basic arithmetic through calculus, study advanced topics in mathematics education and leadership, re-examine curriculum and instruction based on their VMI learning, and complete their school-based research projects and share findings with colleagues. Additionally, mini-workshops presented by the VMI instructional staff will provide further mathematics content enrichment that draws upon teachers' prior VMI course experience. Teachers will complete their action research projects and share findings with colleagues.

Appendix F

A table of the units (in bold) from the following courses: *Mathematics as a Second Language (MSL)*; *Functions, Algebra, & Geometry, I (FA&GI)*; *Probability, Measurement, & Geometry (PM&G)*; *Number Theory (NT)*; and topics comprised in each unit.

MSL: Fundamentals
Expressions vs. equations.
The meaning of equal.
Arithmetic, algebra, and geometry are inseparable.
Math is problem solving.
Solving equations.

Multiple representations of a solution.

Processes and inverse processes.
Manipulations vs. abstract thinking.
Arithmetic: rules vs. logical reasoning.

MSL: Place Value and Numeration

Numeration schemes.

Tally marks and the geometry of numbers.
The importance of zero.

Decimal expansions.

Symmetry and left vs. right and orientation.
Efficient algorithms.
The role of rote memorization.
Estimation and rounding off.
Scientific notation.
MSL: Perspectives on Addition

Tally marks and counting.
Properties of addition.
Adjective-noun theme for addition.

Addition with fractions.

Geometry of addition.

Problem solving with addition.
Applications to algebra.
Applications of the adjective-noun theme to place value, addition with fractions, and algebraic expressions.

Addition of signed numbers.
MSL: Subtraction: The Art of Unadding

Subtraction as addition.
Adjective-noun theme for subtraction.

Subtraction as algebra.

Problem solving with subtraction

Addition and subtraction as inverse processes.
Signed numbers.
Models for subtraction.

Models for signed numbers.
MSL: Multiplication, Area, and the Pythagorean Theorem
What is multiplication?
Multiplication as repeated addition.
Multiplication with the area model.
The distributive property.
The Pythagorean Theorem.
Problem solving with multiplication.

The special role of 1 in multiplication.
Repeated multiplication: exponential notation.

MSL: The Many Faces of Division

Division using multiplication.

The adjective-noun theme for division.

The fraction form of division.
Multiplication and division as inverse processes.
Rules of sign for division.
Problem solving with division.
Rates.

Repeated addition for division.

Partative and quotitive division.

MSL: Fractions

What is a fraction?

Equivalent fractions.

Adjective-noun theme for fractions.

Area model for fractions.

Rates and fractions.

Fractions using algebra.

Cancellation property of division.
Adding and subtracting fractions.

Multiple representations of all operations for fractions.

Multiplying and dividing fractions.

Estimating fractions.
Fractions as decimals and percents.
Reciprocals.
Improper fractions and mixed numbers.
Long division.

MSL: Rates and Conversion
Conversion and inverse conversion.
Constant rates and proportion.
Unit comparisons.
Inverse relation.
Graphical approximations.
Rate to ratio.
Graph of a proportion
MSL: Rational, Irrational, and Real Numbers
Decimals.
Mixed numbers and decimals.
Terminating and non-terminating decimals.
Converting decimals to fractions.
Real numbers.
Rational and irrational numbers.
Problem solving with rational and irrational numbers.
An abstract approach to arithmetic.
FA&GI: The Function Concept
Processes and inverse processes continued.
Inverse functions.
Distance functions (absolute value).
Reciprocal function: indirect variation.
FA&GI: Proportion and Linear Relationships
Conversion and inverse conversion.
Graphing the inverse.
Problem solving with conversions.
The conversion of rates.
Problem solving with rates.
The chain rule for related rates.
Rates to ratios.
FA&GI: Slope and Straight Lines

Linear equations.
Slope-intercept form.
Calculating slope.
Straight lines.
Lines: their equations and graphs.
Inequalities.
Parallel lines and proportion.
Problem solving with slope and straight lines.
Slope as a rate.
Slope and similar triangles.
Slope and parallel lines.
FA&GI: Functions
Conversion in function notation.
Domain and range.
One-to-one and onto.
Problem solving with functions.
Composition of functions.
FA&GI: Linear Function Problems
Inverse functions.
Rate functions.
Problem solving with functions.
FA&GI: Systems of Equations
System of equations.
Simultaneous equations.
Systems of equations in three variables.
Simultaneous equations.
FA&GI: Introduction to Nonlinear Functions
Linear vs. nonlinear functions.
Quadratic functions.
Step functions.
Piecewise linear functions.
Reciprocal functions.
Distance functions.
PM&G: Measurement
Area
Perimeter
Length
Volume
PM&G: Probability
Conditional probability
Expected value
Experimental probability
Theoretical probability
PM&G: Geometry
Polyhedrons
Solid shapes
Measuring volume
Angular measurement
Parallel lines

Similar triangles
Eulers formula
NT: Properties of Positive Integers with Respect to Multiplication and Division
Prime numbers.
Composite numbers.
Factoring.
Relatively prime numbers.
Fundamental Theorem of Arithmetic.
Greatest common factor.
Least common multiple.
Least common denominator.
Division algorithm.
Divisibility rules.
Prime numbers, GCF, LCM, and fractions.
Linear functions, slope, and factorization.
NT: Number Systems
Base 10 and expanded notation.
Scientific notation.
Computing in different bases.
The history of number systems.
NT: Counting Problems
Combinations.
Permutations.

Appendix G

Data from six VMI courses over ten years was inputted into a Statistical Analysis System (SAS) program for analysis. Below is the SAS program followed by a summary of the data and then the output from the program.

SAS Program

```
proc means DATA = good;
TITLE 'Data from all Courses and all Years';
run;
proc univariate DATA=good;
VAR Diff;
run;
DATA THESIS.test;
IF Course= 'Calculus';
run;
proc print DATA=THESIS.test;
TITLE 'Calc Results';
run;
*Read in test data and print a table;
proc print DATA=THESIS.test;
run;
*Sort data by class and then by difference and rename sorted data;
proc sort DATA=THESIS.test OUT=THESIS.good;
BY Class Diff;
*Print sorted data, eliminating pre and post scores (just printing the
differences);
proc print DATA = THESIS.good;
VAR ID Cohort Class Diff;
*Add a title to the output;
TITLE 'testing tesing does this work';
run;
*Analyze all data together;
proc means DATA = THESIS.good;
TITLE 'Data from all Courses and all Years';
run;
proc univariate DATA=THESIS.good;
VAR Diff;
run;

*Frequency table to show the distribution of categorical data values
(Cohort and VMI course);
proc freq DATA=THESIS.good;
TABLES Class Cohort Class*Cohort ID;
*see page 244 for things like chisq conf limits Fishers exact test,
etc;
run;

*This makes an a graph of the frequency of the differences across the
courses;
proc sgplot DATA=THESIS.good;
VBAR Class/ GROUP=Diff;
run;
```

```
*Maybe a more useful graph: it shows the mean percent of improvement
across the different VMI courses;
proc sgplot DATA=THESIS.good;
VBAR Class/ RESPONSE=Diff STAT=MEAN;
run;
*Box plots for the differences between pre/post scores across course;
proc sgplot DATA=THESIS.good;
VBOX Diff / CATEGORY=Class;
TITLE 'Differences in Pre/Post Scores Across Courses';
run;



*Take only Calc data;
DATA THESIS.calc;
SET THESIS.good;
IF Class= 'Calculus';
run;

proc sort DATA=THESIS.calc OUT=THESIS.goodcalc;
BY Diff;

proc print DATA=THESIS.goodcalc;
VAR ID Cohort Class Diff;
TITLE 'Calc Results';
run;
proc univariate DATA=THESIS.goodcalc;
VAR Diff;
*TITLE;
*CDFPLOT; *requests a cumulative distribution function plot;
*HISTOGRAM; *requests a histogram;
*PPLOT; *requests a probability-probability plot;
*PROBPLOT; *requests a probability plot;
*QQPLOT; *requests a quantile-quantile plot;
run;
*A histogram for the improvement in pre/post scores. There are two
density distributions
overlaid on the histogram-the solid line represents the normal
distribution and the dotted
line represents the kernel density estimate;
proc sgplot DATA=THESIS.goodcalc;
HISTOGRAM Diff / SHOWBINS;
DENSITY Diff;
DENSITY Diff / TYPE = KERNEL;
run;

*FOR ONE COURSE ACROSS THE YEARS THIS SHOULD LOOK BETTER WHEN THERE IS
MORE THAN ONE YEAR FOR A COURSE see pg 236 for more;
proc sgplot DATA = THESIS.goodcalc;
REG X=Cohort Y=Diff; *mean diff here of all?;
YAXIS LABEL = 'Difference in Pre/Post Score';
TITLE "Differences in Scores Across the Years";
run;
```

```
*first trial;
*read in data and print a table;
proc print data=THESIS.mpg2007;
RUN;


*sort data by difference;
proc sort DATA=THESIS.mpg2007 OUT=Smpg;
BY Diff;
proc print DATA=Smpg;
TITLE 'Data Sorted by Change in Pre/Port Scores';
run;

proc means DATA=Smpg;
run;
```

Summary of Data Results

6 variables
-ID
-Cohort (year)
-Class (VMI course)
-Pre (percentage on pretest)
-Post (percentage on posttest)
-Diff (difference between pre and post scores)

761 values

Data from all courses and all cohorts:
Mean pre-test score: 46.31 (min 0, max 100)
Mean post-test score: 71.50 (min 6.90, max 100)

Difference from pre-test to post-test score:
Mean: 25.19 (Percentage improvement from pre-test to post-test for an individual).
Median: 24.14
Std Deviation 18.83
Skewness 0.29
Student's t test: t=36.89 Pr>abs (t) is <.001
Quantiles (for improvement):
100% - 82.95
95% - 58.62
75% - 37.93
50% - 24.14
25% - 10.34
10% - 2.78
5% - (-2.78)

0% - (-17.65)

Mean, median and mode for percent increase from pre to post test in each course:

Calculus:
- Mean 44.39
- Median 46.43
- Mode 41.38

NT
- Mean 31.39
- Median 31.03
- Mode 34.49

FA&G II
- Mean 30.61
- Median 29.17
- Mode 8.33

FA&G I
- Mean 22.66
- Median 23.81
- Mode 0

FA&G III
- Mean 22.29
- Median 20.69
- Mode 10.34

PM&G
- Mean 19.81
- Median 20.83
- Mode 12.50

MSL
- Mean 10.87
- Median 8.82
- Mode 0

Courses without enough data

Stat I
Stat II
Stat III
Calculus II
Capstone

Data from SAS:

Data From All Courses and All Years

The MEANS Procedure

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
Cohort	**Cohort**	761	2007.61	2.2521531	2002.00	2012.00
Pre	**Pre**	761	46.3107005	25.7473962	0	100.0000000
Post	**Post**	761	71.4960090	19.8688232	6.8965517	100.0000000
Diff	**Diff**	761	25.1852690	18.8331322	-17.6470588	82.9545455

Analysis of VMI Data

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	761	**Sum Weights**	761
Mean	25.185269	**Sum Observations**	19165.9897
Std Deviation	18.8331322	**Variance**	354.68687
Skewness	0.28971353	**Kurtosis**	-0.6382142
Uncorrected SS	752262.629	**Corrected SS**	269562.021
Coeff Variation	74.7783644	**Std Error Mean**	0.68270051

Basic Statistical Measures			
Location		**Variability**	
Mean	25.18527	**Std Deviation**	18.83313
Median	24.13793	**Variance**	354.68687
Mode	0.00000	**Range**	100.60160
		Interquartile Range	27.58621

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	36.89066	**Pr > |t|**	<.0001
Sign	**M**	323.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	130713	**Pr >= |S|**	<.0001

Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	82.95455
99%	66.66667

Quantiles (Definition 5)	
Quantile	**Estimate**
95%	58.62069
90%	51.85185
75% Q3	37.93103
50% Median	24.13793
25% Q1	10.34483
10%	2.77778
5%	-2.77778
1%	-10.00000
0% Min	-17.64706

Extreme Observations

Lowest		**Highest**
Value	**Obs**	**Value**
-17.6471	676	67.8571
-14.2857	250	67.8571
-14.2857	240	69.4444
-14.2857	219	70.3704
-13.6364	647	82.9545

Data from all Cou

The MEANS Procedure

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
Cohort	**Cohort**	761	2007.61	2.2521531	2002.00	2012.00
Pre	**Pre**	761	46.3107005	25.7473962	0	100.0000000
Post	**Post**	761	71.4960090	19.8688232	6.8965517	100.0000000
Diff	**Diff**				-17.6470588	

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
		761	25.1852690	18.8331322	8	82.9545455

Data from all Courses and all Years

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	761	**Sum Weights**	761
Mean	25.185269	**Sum Observations**	19165.9897
Std Deviation	18.8331322	**Variance**	354.68687
Skewness	0.28971353	**Kurtosis**	-0.6382142
Uncorrected SS	752262.629	**Corrected SS**	269562.021
Coeff Variation	74.7783644	**Std Error Mean**	0.68270051

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Location		**Variability**	
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Median	24.13793	**Variance**	354.68687
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		Interquartile Range	27.58621

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	36.89066	**Pr > |t|**	<.0001
Sign	**M**	323.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	130713	**Pr >= |S|**	<.0001

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Quantile	**Estimate**
100% Max	82.95455
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Quantiles (Definition 5)	
Quantile	**Estimate**
95%	58.62069
90%	51.85185
75% Q3	37.93103
50% Median	24.13793
25% Q1	10.34483
10%	2.77778
5%	-2.77778
1%	-10.00000
0% Min	-17.64706

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
-17.6471	419	67.8571	45
-14.2857	189	67.8571	46
-14.2857	188	69.4444	497
-14.2857	187	70.3704	94
-13.6364	234	82.9545	275

Data from all Courses and all Years

The FREQ Procedure

Class

Class	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Calculus	154	20.24	154	20.24
FAGI	79	10.38	233	30.62
FAGII	61	8.02	294	38.63
FAGIII	86	11.30	380	49.93
MSL	174	22.86	554	72.80
NT	63	8.28	617	81.08
PMG	130	17.08	747	98.16
Statistics	14	1.84	761	100.00

Cohort

Cohort	Frequency	Percent	Cumulative Frequency	Cumulative Percent
2002	38	4.99	38	4.99
2004	48	6.31	86	11.30
2005	22	2.89	108	14.19
2006	111	14.59	219	28.78
2007	112	14.72	331	43.50
2008	100	13.14	431	56.64
2009	200	26.28	631	82.92
2010	86	11.30	717	94.22
2011	24	3.15	741	97.37
2012	20	2.63	761	100.00

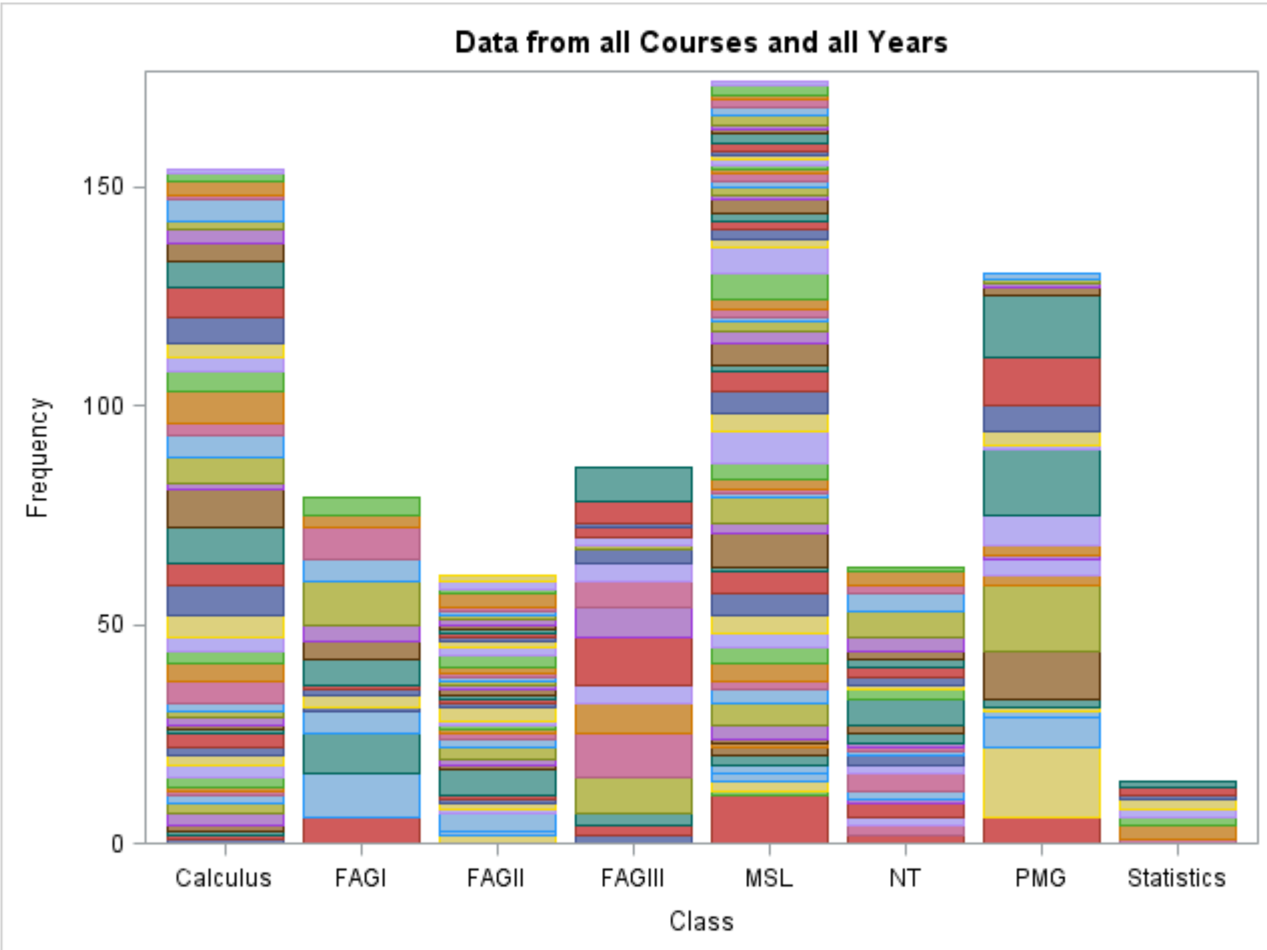
Frequency
Percent
Row Pct
Col Pct

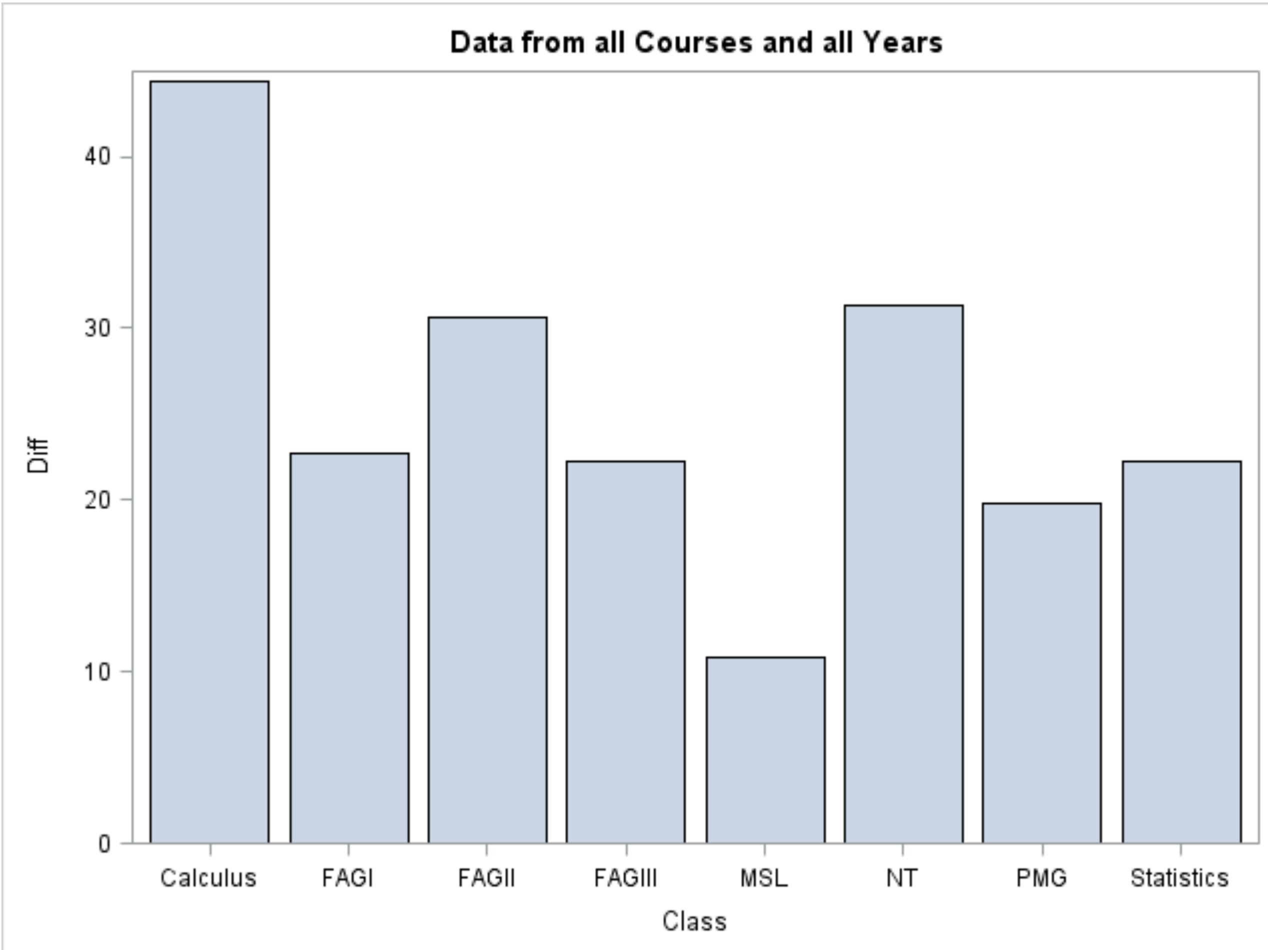
Table of Class by Cohort

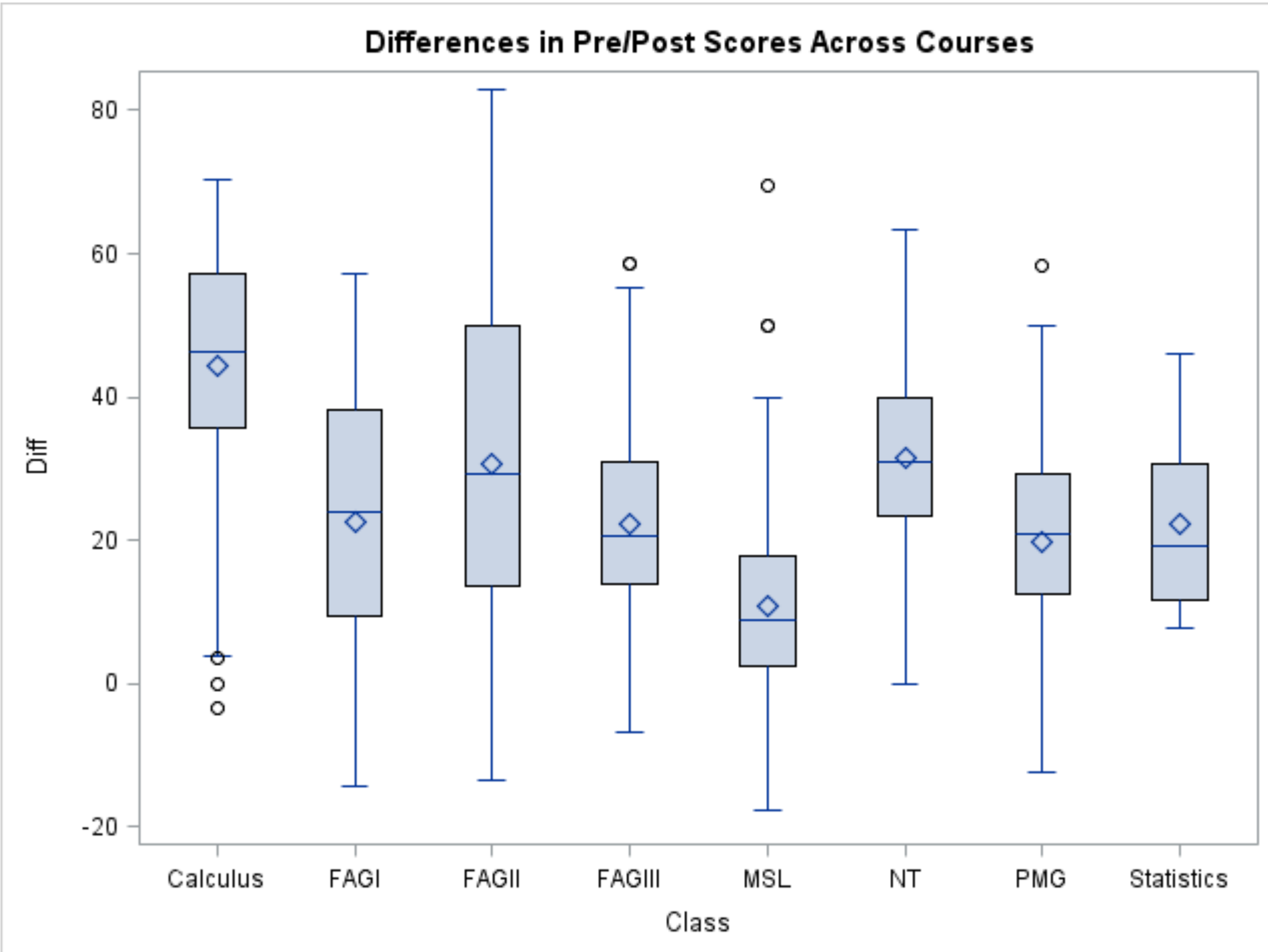
Class(Class)	Cohort(Cohort)										
	2002	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**Total**
Calculus	0 0.00 0.00 0.00	24 3.15 15.58 50.00	22 2.89 14.29 100.00	22 2.89 14.29 19.82	26 3.42 16.88 23.21	18 2.37 11.69 18.00	42 5.52 27.27 21.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	154 20.24
FAGI	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	23 3.02 29.11 20.72	0 0.00 0.00 0.00	9 1.18 11.39 9.00	35 4.60 44.30 17.50	12 1.58 15.19 13.95	0 0.00 0.00 0.00	0 0.00 0.00 0.00	79 10.38
FAGII	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	22 2.89 36.07 19.82	0 0.00 0.00 0.00	20 2.63 32.79 20.00	0 0.00 0.00 0.00	19 2.50 31.15 22.09	0 0.00 0.00 0.00	0 0.00 0.00 0.00	61 8.02
FAGIII	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	25 3.29 29.07 22.32	17 2.23 19.77 17.00	44 5.78 51.16 22.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	86 11.30

MSL	38 4.99 21.84 100.00	24 3.15 13.79 50.00	0 0.00 0.00 0.00	21 2.76 12.07 18.92	0 0.00 0.00 0.00	0 0.00 0.00 0.00	34 4.47 19.54 17.00	13 1.71 7.47 15.12	24 3.15 13.79 100.00	20 2.63 11.49 100.00	174 22.86
NT	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	24 3.15 38.10 21.43	18 2.37 28.57 18.00	0 0.00 0.00 0.00	21 2.76 33.33 24.42	0 0.00 0.00 0.00	0 0.00 0.00 0.00	63 8.28
PMG	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	23 3.02 17.69 20.72	23 3.02 17.69 20.54	18 2.37 13.85 18.00	45 5.91 34.62 22.50	21 2.76 16.15 24.42	0 0.00 0.00 0.00	0 0.00 0.00 0.00	130 17.08
Statistics	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	14 1.84 100.00 12.50	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	0 0.00 0.00 0.00	14 1.84
Total	38 4.99	48 6.31	22 2.89	111 14.59	112 14.72	100 13.14	200 26.28	86 11.30	24 3.15	20 2.63	761 100.00

	14.59		13.14	26.28	




Data from all Courses and all Years
Diff
0
10
20
30
40
Calculus
FAGI
FAGII
FAGIII
MSL
NT
PMG
Statistics
Class



MSL Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	174	**Sum Weights**	174
Mean	10.8711153	**Sum Observations**	1891.57406
Std Deviation	13.613578	**Variance**	185.329505
Skewness	0.98852723	**Kurtosis**	1.60875693
Uncorrected SS	52625.5241	**Corrected SS**	32062.0043

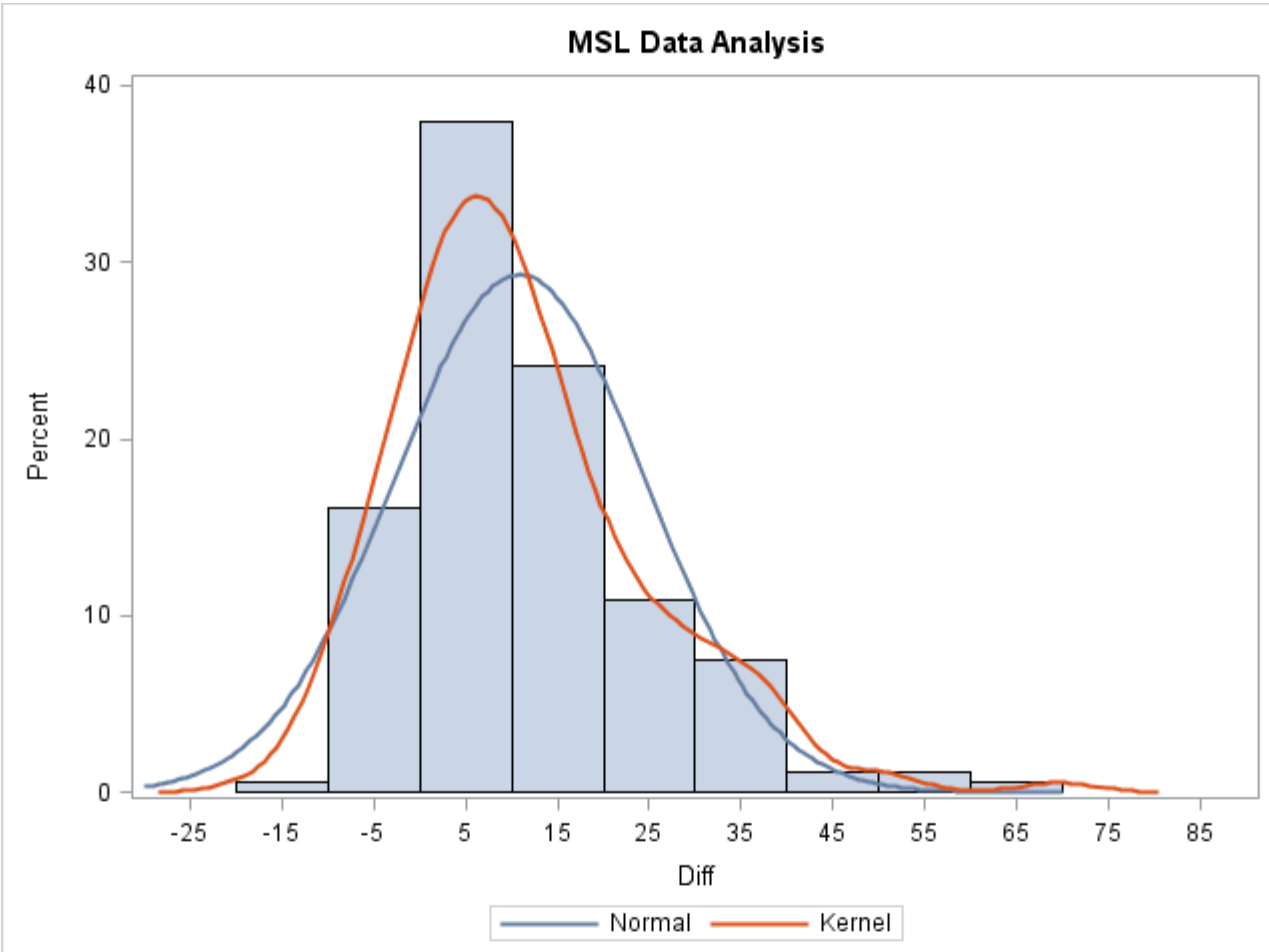
Moments			
Coeff Variation	125.227059	**Std Error Mean**	1.03204268

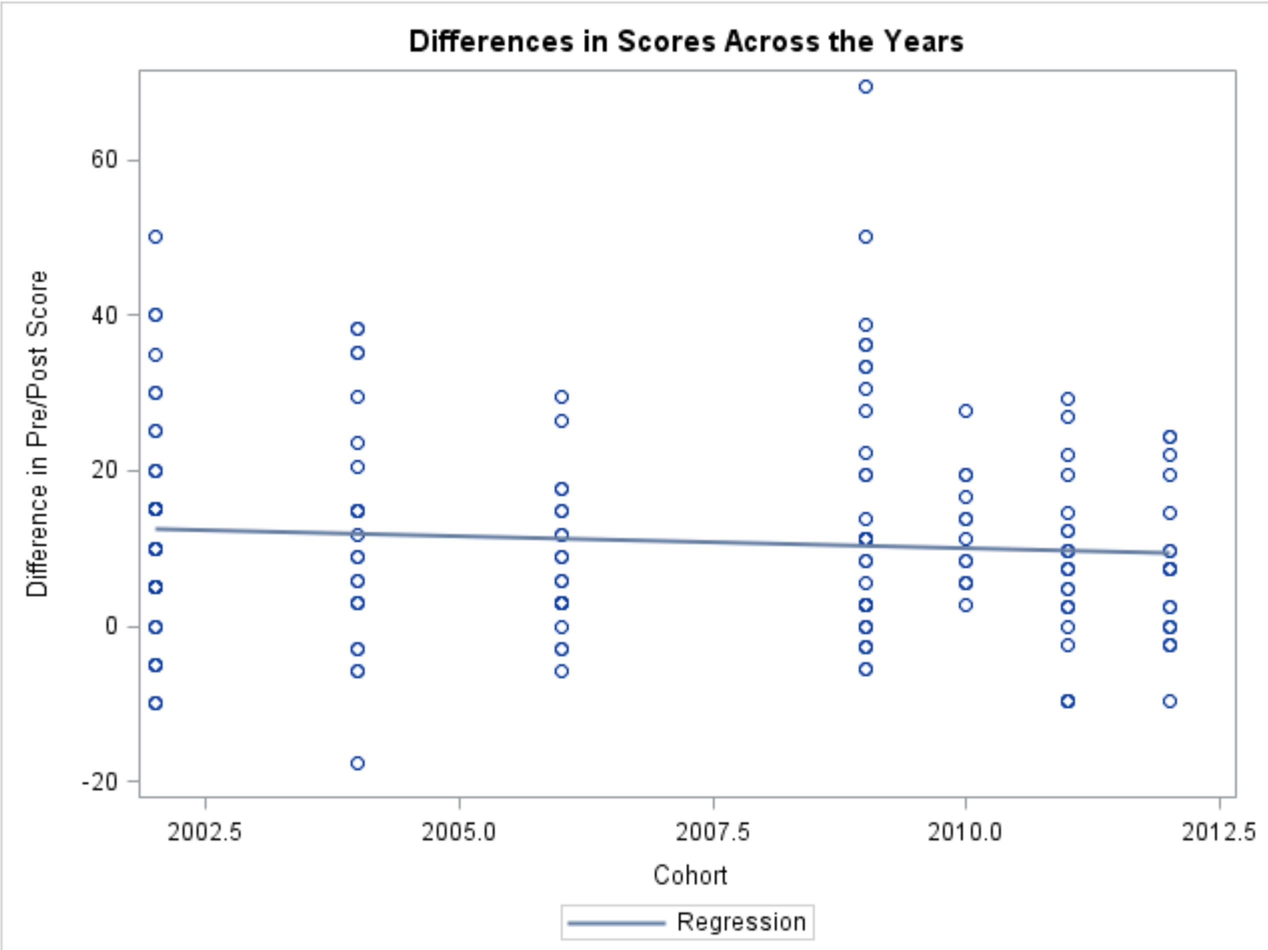
Basic Statistical Measures			
Location		**Variability**	
Mean	10.87112	**Std Deviation**	13.61358
Median	8.82353	**Variance**	185.32950
Mode	0.00000	**Range**	87.09150
		Interquartile Range	15.20803

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	10.53359	**Pr > |t|**	<.0001
Sign	**M**	52.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	5391	**Pr >= |S|**	<.0001

Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	69.44444
99%	50.00000
95%	36.11111
90%	30.00000
75% Q3	17.64706
50% Median	8.82353
25% Q1	2.43902
10%	-5.00000
5%	-9.75610
1%	-10.00000
0% Min	-17.64706

Extreme Observations			
Lowest		Highest	
Value	Obs	Value	Obs
-17.6471	1	40.0000	170
-10.0000	4	40.0000	171
-10.0000	3	50.0000	172
-10.0000	2	50.0000	173
-9.7561	5	69.4444	174


MSL Data Analysis
Percent
40
30
20
10
0
-25
-15
-5
5
15
25
35
45
55
65
75
85
Diff
Normal
Kernel



Functions, Algebra and Geometry I Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	79	**Sum Weights**	79
Mean	22.6642556	**Sum Observations**	1790.47619
Std Deviation	17.4916223	**Variance**	305.956852
Skewness	-0.118513	**Kurtosis**	-0.6312244
Uncorrected SS	64444.4444	**Corrected SS**	23864.6345
Coeff Variation	77.1771315	**Std Error Mean**	1.96796127

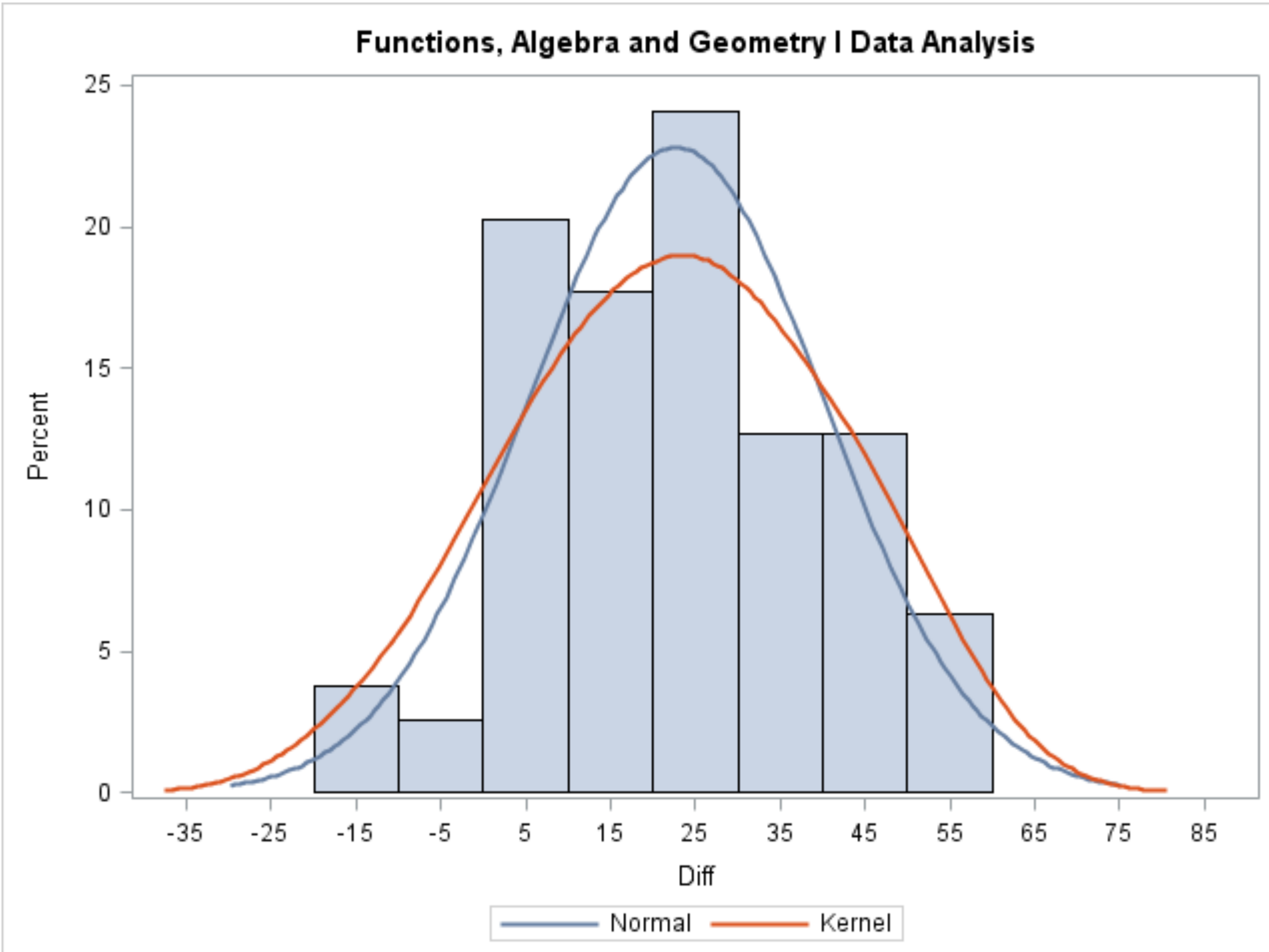
Basic Statistical Measures			
Location		**Variability**	
Mean	22.66426	**Std Deviation**	17.49162
Median	23.80952	**Variance**	305.95685
Mode	0.00000	**Range**	71.42857
		Interquartile Range	28.57143

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	11.51662	**Pr > |t|**	<.0001
Sign	**M**	31.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	1294.5	**Pr >= |S|**	<.0001

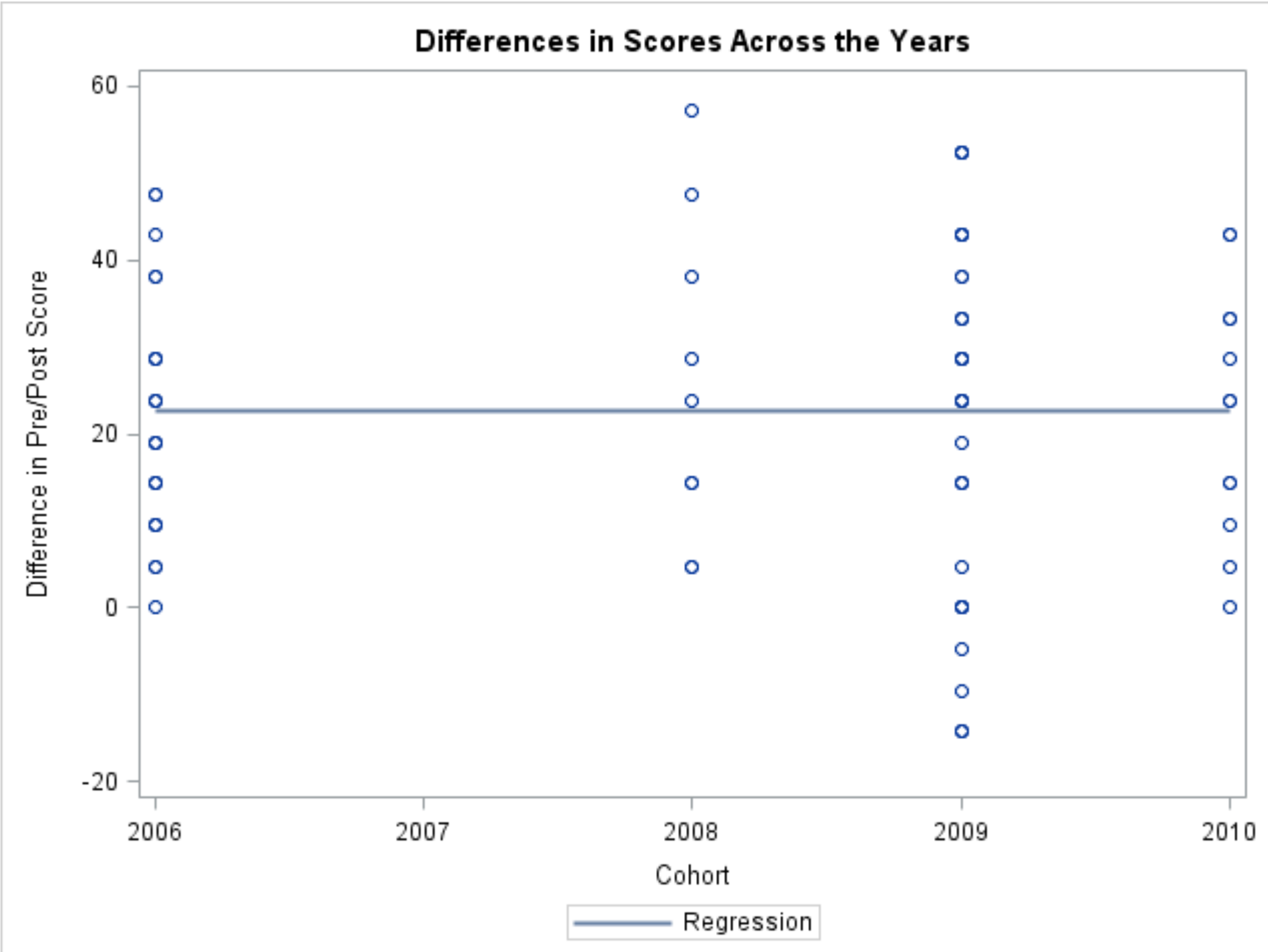
Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	57.14286
99%	57.14286
95%	52.38095
90%	47.61905
75% Q3	38.09524
50% Median	23.80952
25% Q1	9.52381
10%	0.00000
5%	-9.52381
1%	-14.28571
0% Min	-14.28571

Extreme Observations	
Lowest	**Highest**

Value	Obs	Value	Obs
-14.28571	3	52.3810	75
-14.28571	2	52.3810	76
-14.28571	1	52.3810	77
-9.52381	4	52.3810	78
-4.76190	5	57.1429	79



Functions, Algebra and Geometry I Data Analysis
Percent
0
5
10
15
20
25
-35
-25
-15
-5
5
15
25
35
45
55
65
75
85
Diff
Normal
Kernel



Functions, Algebra and Geometry II Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	61	**Sum Weights**	61
Mean	30.607084	**Sum Observations**	1867.03212
Std Deviation	19.3052621	**Variance**	372.693145
Skewness	0.247029	**Kurtosis**	-0.3850905

Moments			
Uncorrected SS	79505.9976	**Corrected SS**	22361.5887
Coeff Variation	63.07449	**Std Error Mean**	2.47178553

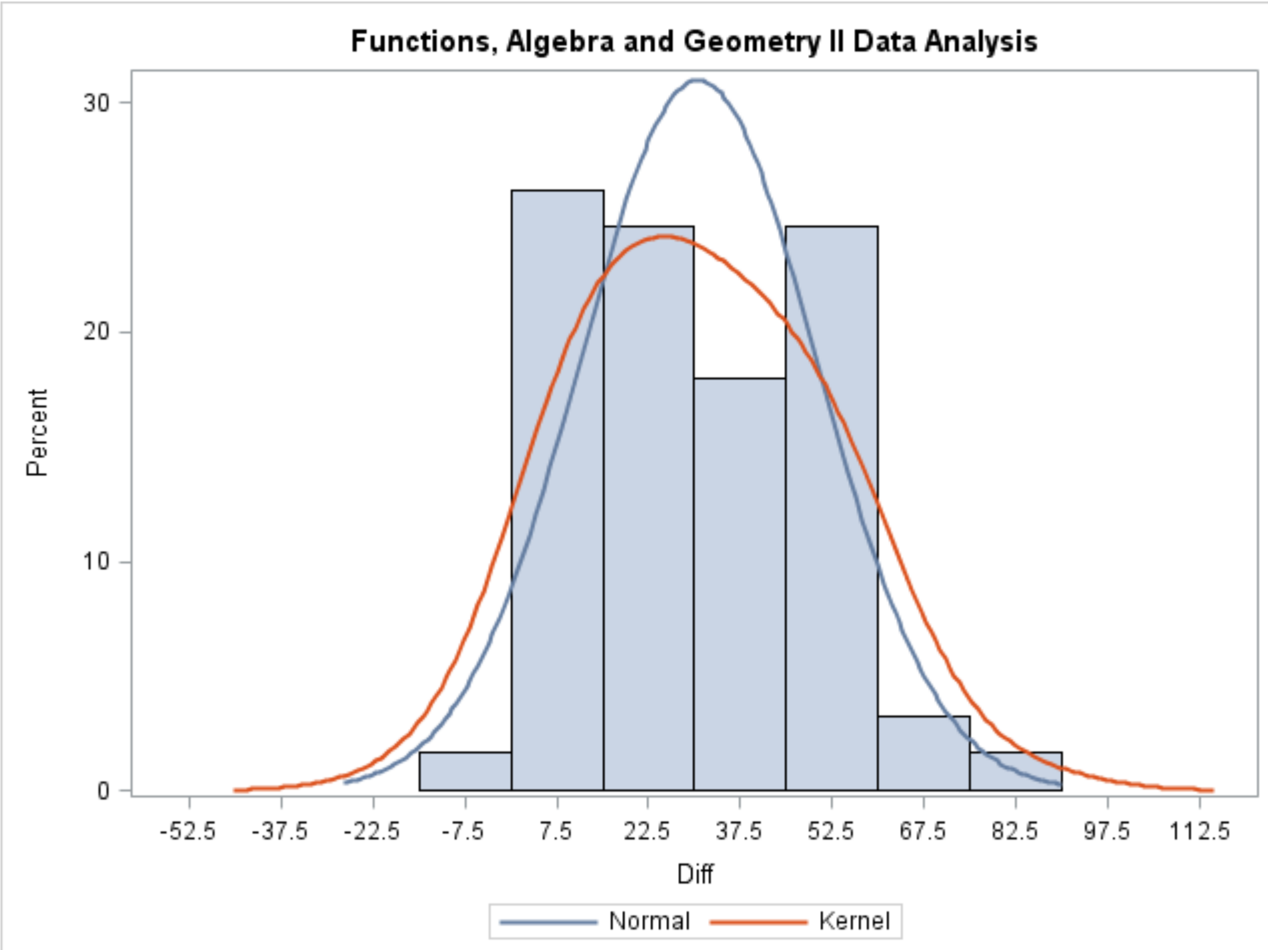
Basic Statistical Measures			
Location		**Variability**	
Mean	30.60708	**Std Deviation**	19.30526
Median	29.17000	**Variance**	372.69315
Mode	8.33000	**Range**	96.59091
		Interquartile Range	36.36364

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	12.38258	**Pr > |t|**	<.0001
Sign	**M**	29.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	930	**Pr >= |S|**	<.0001

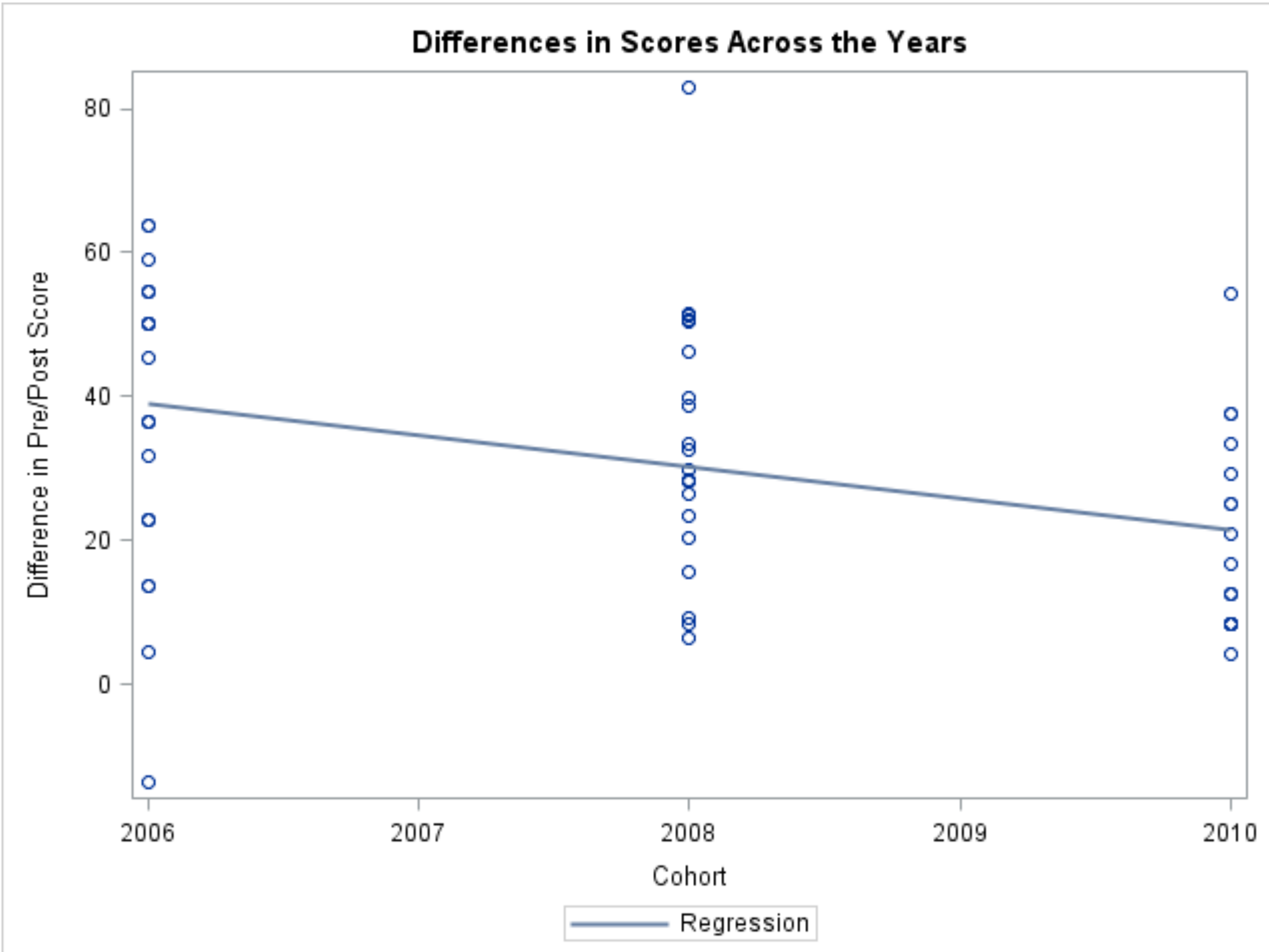
Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	82.95455
99%	82.95455
95%	59.09091
90%	54.54545
75% Q3	50.00000
50% Median	29.17000
25% Q1	13.63636
10%	8.33000
5%	6.43939
1%	-13.63636

Quantiles (Definition 5)	
Quantile	**Estimate**
0% Min	-13.63636

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
-13.63636	1	54.5455	57
4.17000	2	59.0909	58
4.54545	3	63.6364	59
6.43939	4	63.6364	60
8.33000	10	82.9545	61



Functions, Algebra and Geometry II Data Analysis
Percent
30
20
10
0
-52.5
-37.5
-22.5
-7.5
7.5
22.5
37.5
52.5
67.5
82.5
97.5
112.5
Diff
Normal
Kernel



Functions, Algebra and Geometry III Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	86	**Sum Weights**	86
Mean	22.2935044	**Sum Observations**	1917.24138
Std Deviation	14.0214134	**Variance**	196.600035
Skewness	0.43039132	**Kurtosis**	0.17854983
Uncorrected SS	59453.0321	**Corrected SS**	16711.003
Coeff Variation	62.8946135	**Std Error Mean**	1.5119679

Basic Statistical Measures			
Location		**Variability**	
Mean	22.29350	**Std Deviation**	14.02141
Median	20.68966	**Variance**	196.60003
Mode	10.34483	**Range**	65.51724
		Interquartile Range	17.24138

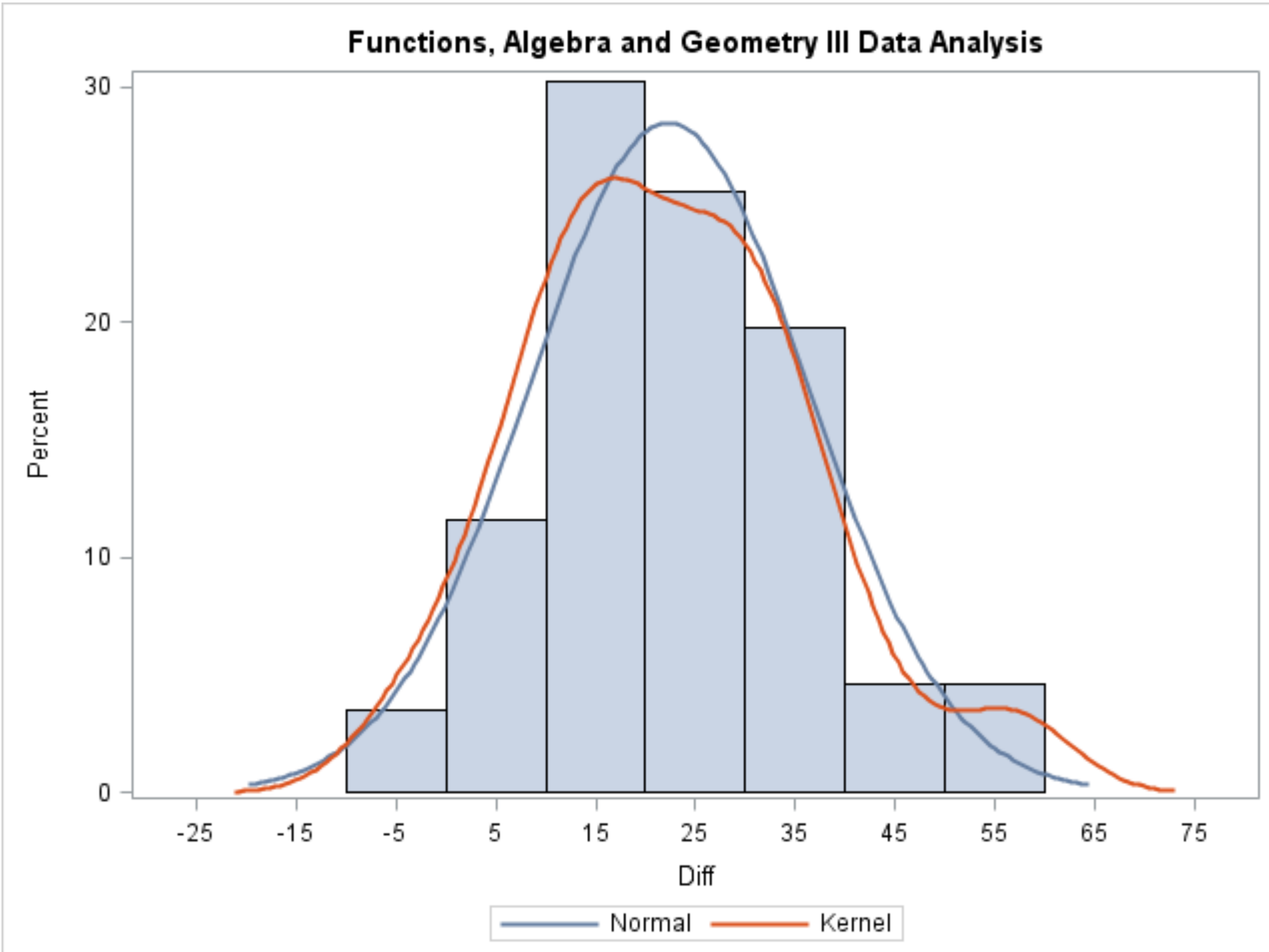
Note: The mode displayed is the smallest of 2 modes with a count of 5.

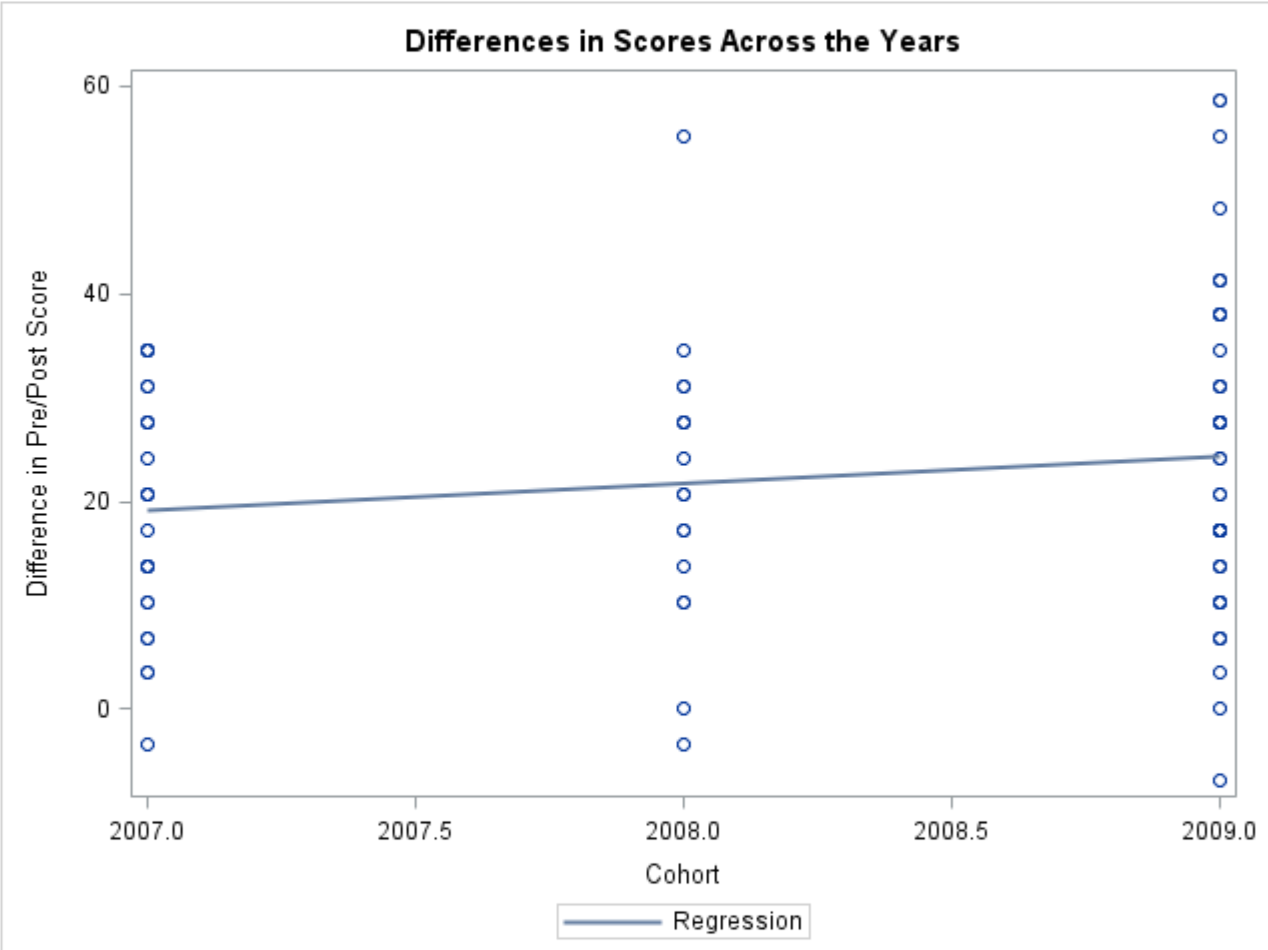
Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	14.74469	**Pr > |t|**	<.0001
Sign	**M**	39	**Pr >= |M|**	<.0001
Signed Rank	**S**	1772	**Pr >= |S|**	<.0001

Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	58.62069
99%	58.62069
95%	48.27586
90%	37.93103
75% Q3	31.03448
50% Median	20.68966
25% Q1	13.79310
10%	6.89655
5%	0.00000
1%	-6.89655
0% Min	-6.89655

Extreme Observations

Lowest		Highest	
Value	**Obs**	**Value**	**Obs**
-6.89655	1	48.2759	82
-3.44828	2	55.1724	83
-3.44828	3	55.1724	84
0.00000	5	58.6207	85
0.00000	4	58.6207	86


Functions, Algebra and Geometry III Data Analysis
Percent
0
10
20
30
-25
-15
-5
5
15
25
35
45
55
65
75
Diff
Normal
Kernel



Probability, Measurement and Geometry Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	130	**Sum Weights**	130
Mean	19.8075897	**Sum Observations**	2574.98667
Std Deviation	12.3326083	**Variance**	152.093227
Skewness	0.1458969	**Kurtosis**	0.08300092

Moments			
Uncorrected SS	70624.3057	**Corrected SS**	19620.0262
Coeff Variation	62.262034	**Std Error Mean**	1.0816413

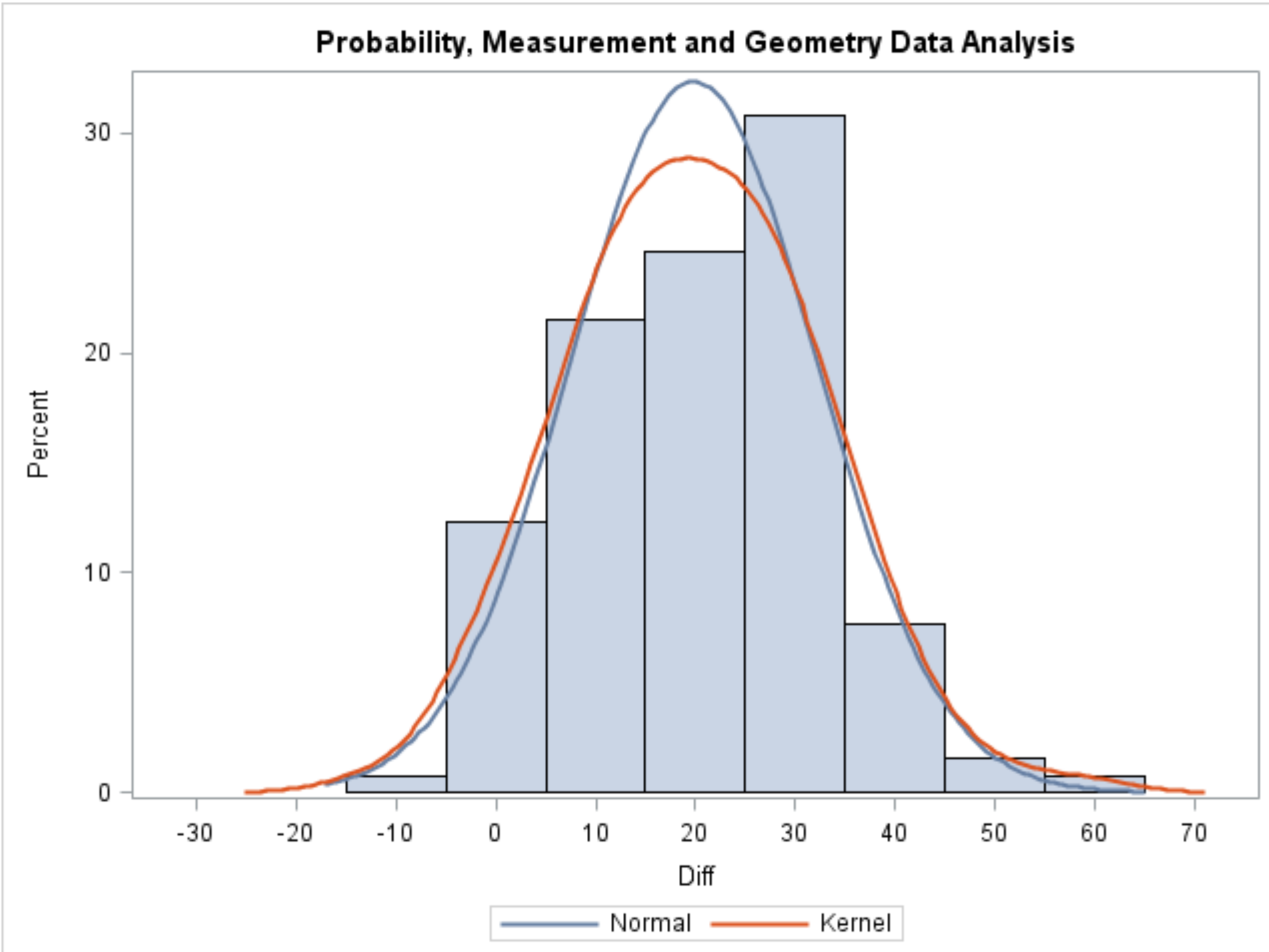
Basic Statistical Measures			
Location		**Variability**	
Mean	19.80759	**Std Deviation**	12.33261
Median	20.83000	**Variance**	152.09323
Mode	12.50000	**Range**	70.83333
		Interquartile Range	16.66667

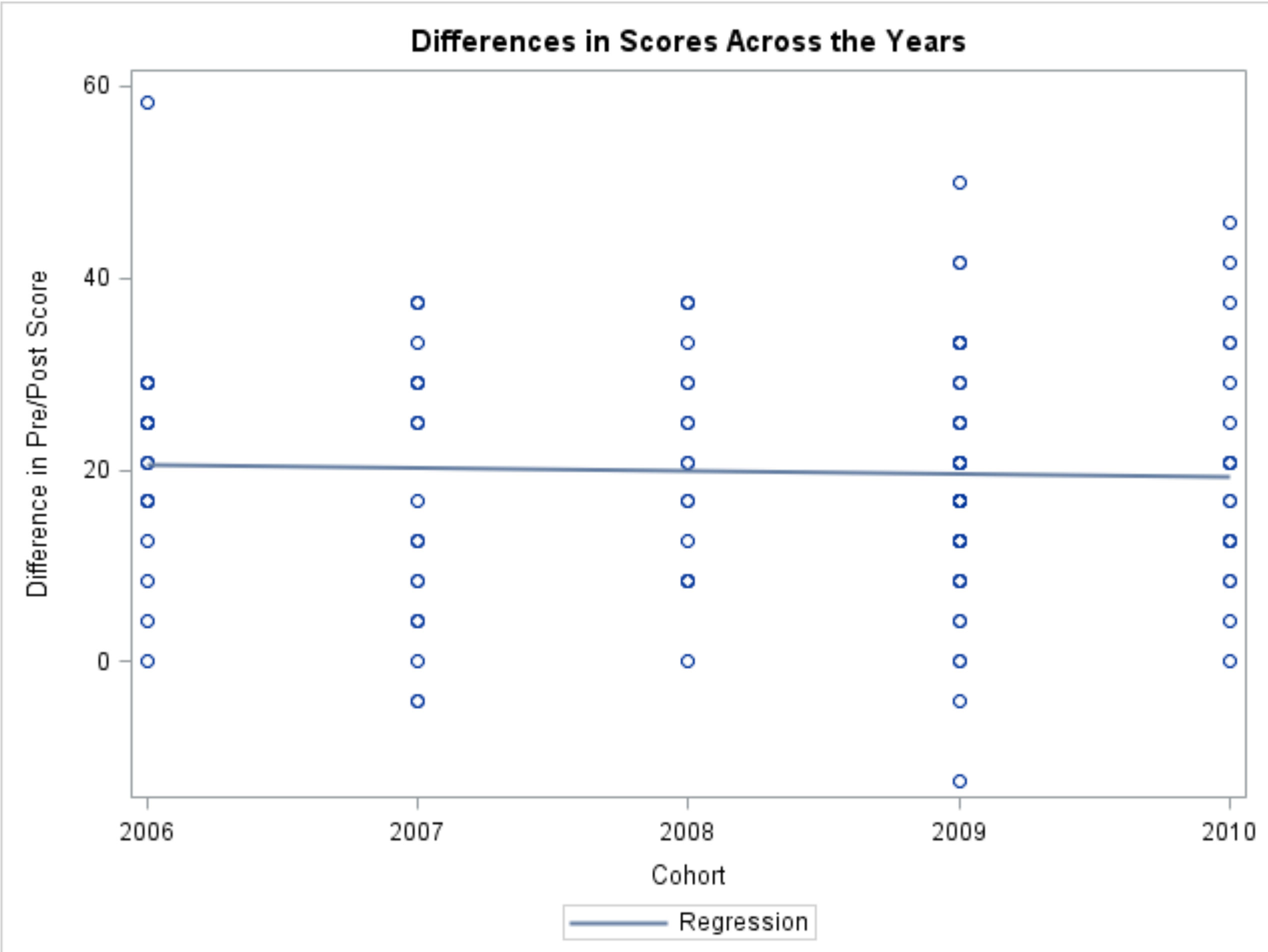
Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	18.31253	**Pr > |t|**	<.0001
Sign	**M**	58	**Pr >= |M|**	<.0001
Signed Rank	**S**	3839.5	**Pr >= |S|**	<.0001

Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	58.33333
99%	50.00000
95%	37.50000
90%	35.41667
75% Q3	29.16667
50% Median	20.83000
25% Q1	12.50000
10%	4.16667
5%	0.00000
1%	-4.16667

Quantiles (Definition 5)	
Quantile	**Estimate**
0% Min	-12.50000

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
-12.50000	1	41.6667	126
-4.16667	2	41.6700	127
-4.16667	3	45.8300	128
-4.16667	4	50.0000	129
0.00000	10	58.3333	130


Probability, Measurement and Geometry Data Analysis
Percent
30
20
10
0
-30
-20
-10
0
10
20
30
40
50
60
70
Diff
Normal
Kernel



Number Theory Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments			
N	63	**Sum Weights**	63
Mean	31.3902573	**Sum Observations**	1977.58621
Std Deviation	14.934936	**Variance**	223.052312
Skewness	0.01730728	**Kurtosis**	-0.081121

Moments			
Uncorrected SS	75906.1831	**Corrected SS**	13829.2433
Coeff Variation	47.5782528	**Std Error Mean**	1.88162507

Basic Statistical Measures			
Location		**Variability**	
Mean	31.39026	**Std Deviation**	14.93494
Median	31.03448	**Variance**	223.05231
Mode	34.48276	**Range**	63.33333
		Interquartile Range	16.66667

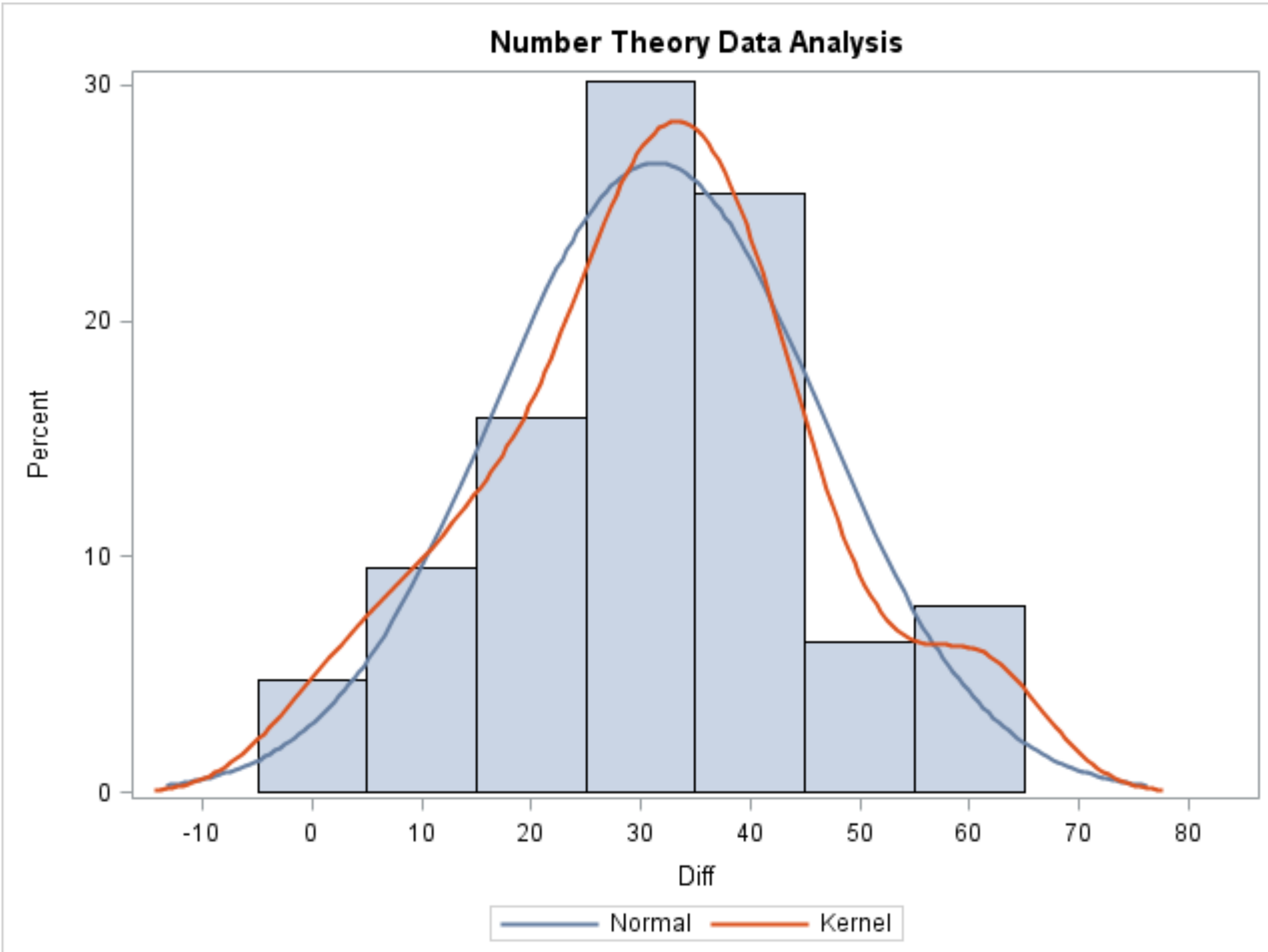
Note: The mode displayed is the smallest of 2 modes with a count of 4.

<table>
<tr><th colspan="5">Tests for Location: Mu0=0</th></tr>
<tr><th>Test</th><th colspan="2">Statistic</th><th colspan="2">p Value</th></tr>
<tr><td>Student's t</td><td>t</td><td>16.68253</td><td>Pr > |t|</td><td><.0001</td></tr>
<tr><td>Sign</td><td>M</td><td>30.5</td><td>Pr >= |M|</td><td><.0001</td></tr>
<tr><td>Signed Rank</td><td>S</td><td>945.5</td><td>Pr >= |S|</td><td><.0001</td></tr>
</table>

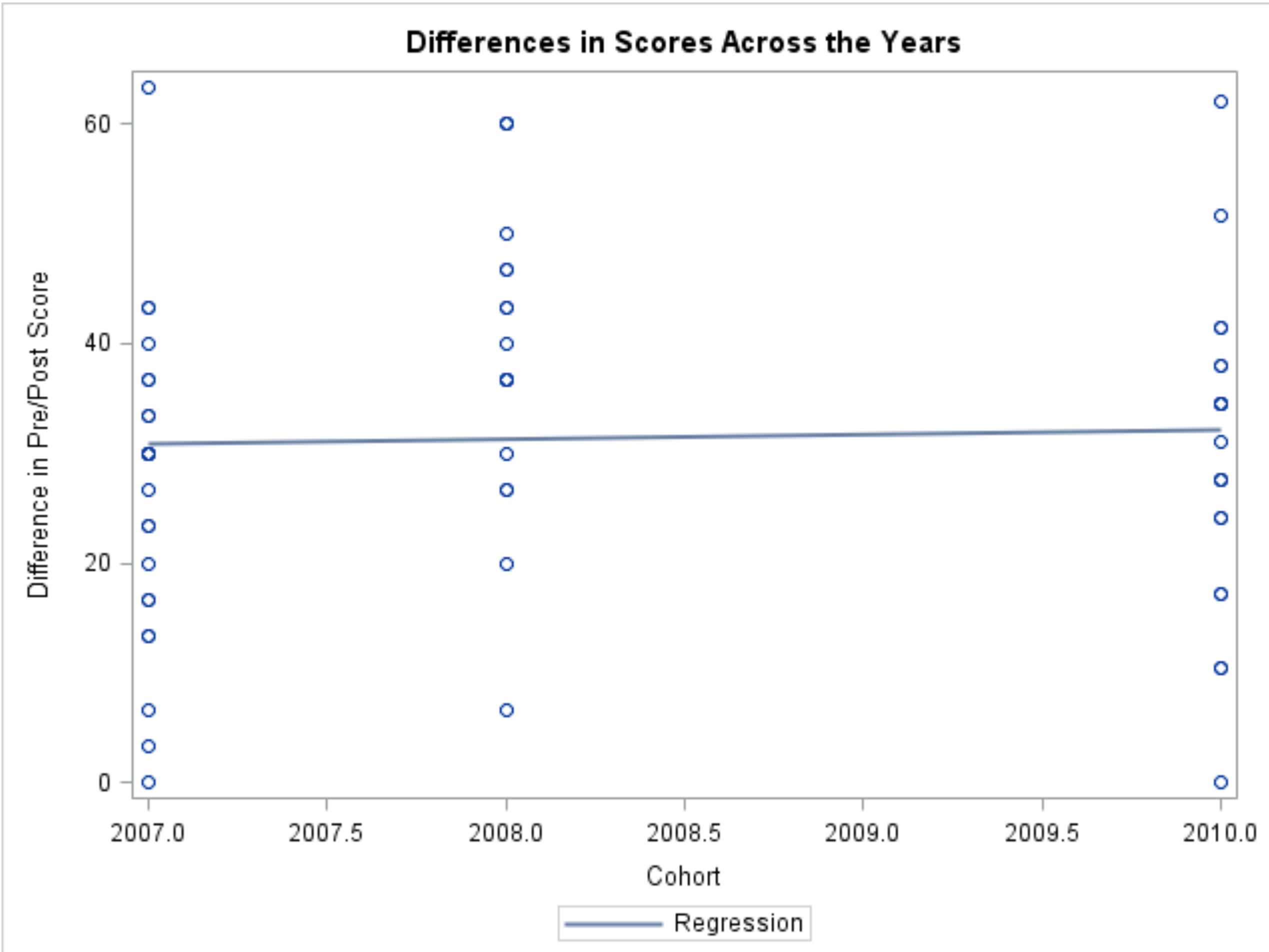
Quantiles (Definition 5)	
Quantile	**Estimate**
100% Max	63.33333
99%	63.33333
95%	60.00000
90%	50.00000
75% Q3	40.00000
50% Median	31.03448
25% Q1	23.33333
10%	10.34483
5%	6.66667

Quantiles (Definition 5)	
Quantile	**Estimate**
1%	0.00000
0% Min	0.00000

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
0.00000	2	60.0000	59
0.00000	1	60.0000	60
3.33333	3	60.0000	61
6.66667	4	62.0690	62
6.66667	5	63.3333	63



Number Theory Data Analysis
Percent
30
20
10
0
-10
0
10
20
30
40
50
60
70
80
Diff
Normal
Kernel


Differences in Scores Across the Years
Difference in Pre/Post Score
0
20
40
60
2007.0
2007.5
2008.0
2008.5
2009.0
2009.5
2010.0
Cohort
Regression

Calculus Data Analysis

The UNIVARIATE Procedure
Variable: Diff (Diff)

Moments

N	154	**Sum Weights**	154
Mean	44.3867185	**Sum Observations**	6835.55464
Std Deviation	15.6974307	**Variance**	246.409332
Skewness	-0.8597943	**Kurtosis**	0.41985338
Uncorrected SS	341108.467	**Corrected SS**	37700.6277
Coeff Variation	35.3651526	**Std Error Mean**	1.26493502

Basic Statistical Measures

Location		Variability	
Mean	44.38672	**Std Deviation**	15.69743
Median	46.42857	**Variance**	246.40933
Mode	41.37931	**Range**	73.81865
		Interquartile Range	21.42857

Note: The mode displayed is the smallest of 2 modes with a count of 6.

Tests for Location: Mu0=0

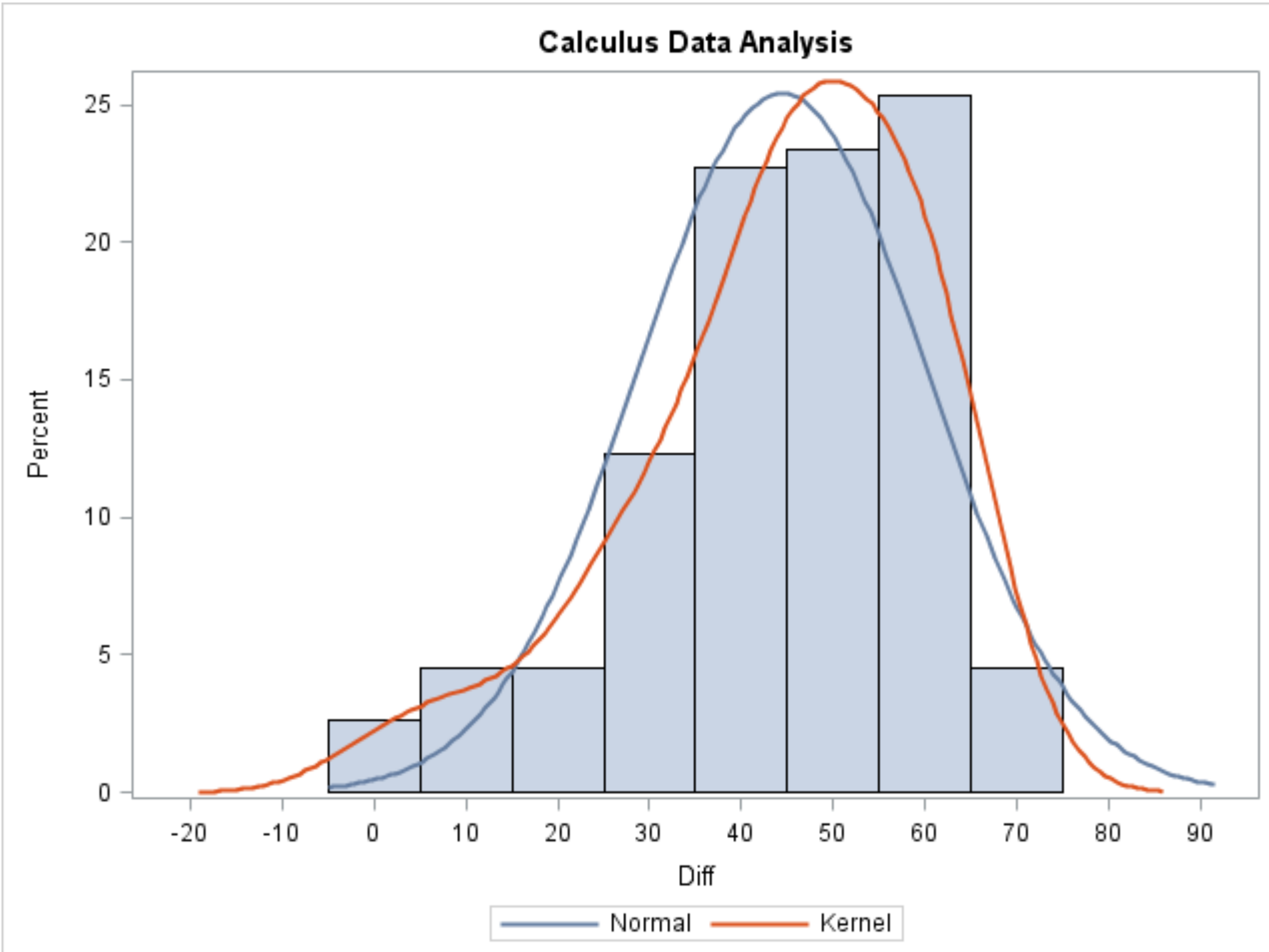
Test	Statistic		p Value	
Student's t	**t**	35.09012	**Pr > |t|**	<.0001
Sign	**M**	75.5	**Pr >= |M|**	<.0001
Signed Rank	**S**	5889	**Pr >= |S|**	<.0001

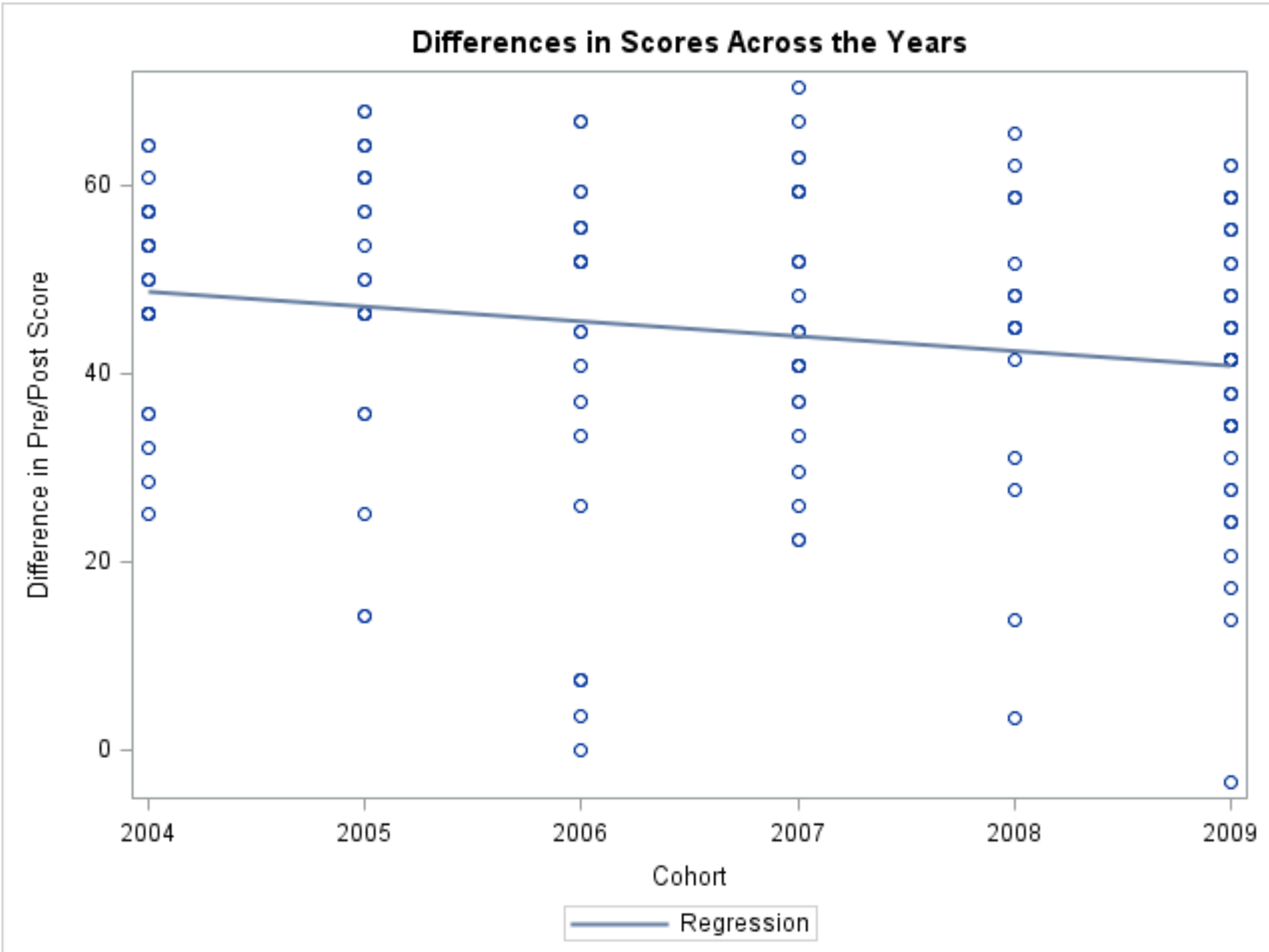
Quantiles (Definition 5)

Quantile	Estimate
100% Max	70.37037

Quantiles (Definition 5)	
Quantile	**Estimate**
99%	67.85714
95%	64.28571
90%	62.06897
75% Q3	57.14286
50% Median	46.42857
25% Q1	35.71429
10%	24.13793
5%	13.79310
1%	0.00000
0% Min	-3.44828

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
-3.44828	1	66.6667	150
0.00000	2	66.6667	151
3.44828	3	67.8571	152
3.70370	4	67.8571	153
7.40741	7	70.3704	154


Calculus Data Analysis
Percent
25
20
15
10
5
0
-20
-10
0
10
20
30
40
50
60
70
80
90
Diff
Normal
Kernel


Differences in Scores Across the Years
Difference in Pre/Post Score
0
20
40
60
2004
2005
2006
2007
2008
2009
Cohort
Regression

Appendix H

Reliability of the Measures

Since the pre and post tests were devised by program faculty , VMI course pre and post tests have been evaluated for reliability of measurement since 2002. Each test was evaluated for reliability of measurement (internal consistency) with a Cronbach alpha coefficient. Pre and post- test scales were analyzed separately and then combined into one scale for the purpose of assessing the reliability. Coefficients ranged from .84 to .96. All measures had sufficient reliability upon which to gauge teacher performance on the knowledge tests.

Examples of the reliability procedure output:

Functions and Algebra
Total Scale

Reliability Statistics Functions and Algebra

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.959	.956	61

Geometry (GMI)
Total Scale

Reliability Statistics GMI

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.938	.937	63

Numbers and Operations
Total Scale

Reliability Statistics Numbers and Operations

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.770	.781	22

Number Theory
Total Scale

Reliability Statistics Number Theory

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.881	.873	50

Probability
Total Scale

Reliability Statistics Probability and Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.831	.834	33

All information in Appendix H came from:

Meyers, H.W., & Harris, D. (2012). *Title IIB Massachusetts Math and Science Partnership Project Evaluation Report of Findings: 2011-2012.* Boston, MA: Lesley University.