One of the simplest conceptual models in quantum statistical physics is a gas of non-interacting particles with bosonic symmetry. In the grand canonical ensemble, particle number and temperature are in equilibrium with an external reservoir and an exact analytical expression can be derived for the partition function at any density and chemical potential. In the canonical ensemble, the total number of particles, N, is fixed and an expression for the partition function can only be generated via a complicated recursion relation.

In this work we apply the recursion formula to obtain the partition function, and thus all thermodynamic quantities, for up to 4 non-interacting bosons in three spatial dimensions at low temperature. Analytical results for the pressure and energy are confirmed using exact path integral quantum Monte Carlo simulations. From here, we turn on weak dipolar interactions between the particles in two spatial dimensions and illustrate how quantum Monte Carlo simulations can be used to explore the relationship between pressure and energy in a system that could be experimentally realized using ultra-cold atomic gases.