

# System Robustness

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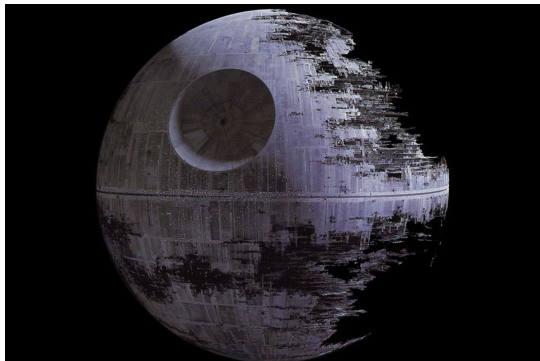


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Robustness  
HOT theory  
Narrative causality  
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## Our emblem of Robust-Yet-Fragile:



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## Robustness

HOT combines things we've seen:

- Variable transformation
- Constrained optimization
- Need power law transformation between variables:  $(Y = X^{-\alpha})$
- Recall PLIPL0 is bad...
- MIWO is good: Mild In, Wild Out
- $X$  has a characteristic size but  $Y$  does not

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## Outline

### Robustness

- HOT theory
- Narrative causality
- Random forests
- Self-Organized Criticality
- COLD theory
- Network robustness

### References



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## Robustness

- System robustness may result from
  - Evolutionary processes
  - Engineering/Design
- Idea: Explore systems optimized to perform under uncertain conditions.
- The handle: 'Highly Optimized Tolerance' (HOT) [4, 5, 6, 10]
- The catchphrase: Robust yet Fragile
- The people: Jean Carlson and John Doyle
- Great abstracts of the world #73: "There aren't any." [7]



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## Robustness

Forest fire example: [5]

- Square  $N \times N$  grid
- Sites contain a tree with probability  $\rho =$  density
- Sites are empty with probability  $1 - \rho$
- Fires start at location  $(i, j)$  according to some distribution  $P_{ij}$
- Fires spread from tree to tree (nearest neighbor only)
- Connected clusters of trees burn completely
- Empty sites block fire
- Best case scenario: Build firebreaks to maximize average # trees left intact given one spark



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## Robustness

- Many complex systems are prone to cascading catastrophic failure: exciting!!!
  - Blackouts
  - Disease outbreaks
  - Wildfires
  - Earthquakes
  - Organisms, individuals and societies
  - Ecosystems
  - Cities
  - Myths: Achilles.
- But complex systems also show persistent robustness (not as exciting but important...)
- Robustness and Failure may be a power-law story...



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## Robustness

- Features of HOT systems: [5, 6]
  - High performance and robustness
  - Designed/evolved to handle known stochastic environmental variability
  - Fragile in the face of unpredicted environmental signals
  - Highly specialized, low entropy configurations
  - Power-law distributions appear (of course...)



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## Robustness

Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test  $D$  ways of adding one tree
- $D =$  design parameter
- Average over  $P_{ij}$  = spark probability
- $D = 1$ : random addition
- $D = N^2$ : test all possibilities

Measure average area of forest left untouched

- $f(c)$  = distribution of fire sizes  $c$  (= cost)
- Yield =  $Y = \rho - \langle c \rangle$



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Specifics:

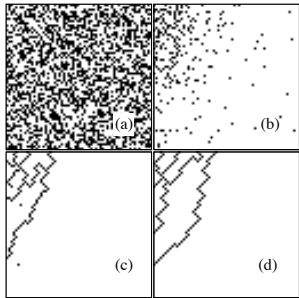
P\_{ij} = P\_{i;a\_x,b\_x} P\_{j;a\_y,b\_y}

where

P\_{i;a,b} \propto e^{-[(i+a)/b]^2}

- In the original work, b\_y > b\_x
Distribution has more width in y direction.

HOT Forests



N = 64

- (a) D = 1
(b) D = 2
(c) D = N
(d) D = N^2

P\_{ij} has a Gaussian decay

- Optimized forests do well on average (robustness)
But rare extreme events occur (fragility)

HOT Forests

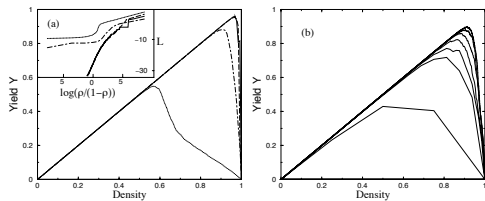


FIG. 2. Yield vs density Y(\rho): (a) for design parameters D = 1 (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with N = 64, and (b) for D = 2 and N = 2, 2^2, ..., 2^7 running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions L = \log(f/(1-f)), on a scale which more clearly differentiates between the curves.

HOT Forests:

Y = 'the average density of trees left unburned in a configuration after a single spark hits.'

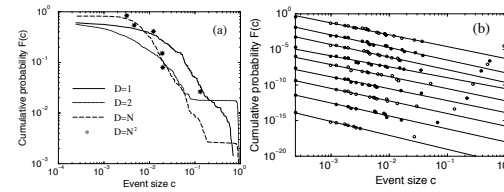


FIG. 3. Cumulative distributions of events F(c): (a) at peak yield for D = 1, 2, N, and N^2 with N = 64, and (b) for D = N^2, and N = 64 at equal density increments of 0.1, ranging at \rho = 0.1 (bottom curve) to \rho = 0.9 (top curve).

Random Forests

D = 1: Random forests = Percolation

- Randomly add trees.
Below critical density \rho\_c, no fires take off.
Above critical density \rho\_c, percolating cluster of trees burns.
Only at \rho\_c, the critical density, is there a power-law distribution of tree cluster sizes.
Forest is random and featureless.

HOT forests nutshell:

- Highly structured
Power law distribution of tree cluster sizes for a broad range of \rho, including below \rho\_c.
No specialness of \rho\_c
Forest states are tolerant
Uncertainty is okay if well characterized
If P\_{ij} is characterized poorly or changes too fast, failure becomes highly likely
Growth is key to toy model which is both algorithmic and physical.
HOT theory is more general than just this toy model.

HOT forests—Real data:

'Complexity and Robustness,' Carlson & Dolye

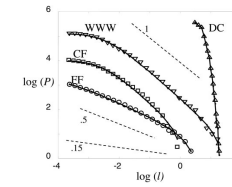


Fig. 5. Log-log (base 10) comparison of DC, WWW, CF, and FF data symbols with PLR models (solid lines) for \rho = 0.0, 0.5, 1.85, or \rho = 1 - (1/\beta) = 1.1, 1.1, 0.854, respectively and the SOC-FF model (\rho = 0.15, dashed). Reference lines of \rho = 0.5, 1 (dotted) are included. The cumulative distributions of frequencies of F(c) in (c) describe the areas burned in the largest 4,384 fires from 1986 to 1995 or all of the U.S. Fire and Wildlife Service large fire (LFI) data, the ~150,000 largest California brushfires from 1978 to 1999 (CB), 150,000 wild fire brushfires at Boston University during 1994 and 1995 (WWW), and code words from DC. The size units: 1,000 km^2 for FF and CF, megahertz (WWW), and bytes (DC) and the logarithmic distribution of the data area chosen for visualization.

HOT theory:

The abstract story, using figurative forest fires:

- Given some measure of failure size y\_i and correlated resource size x\_i with relationship y\_i = x\_i^{-\alpha}, i = 1, ..., N\_sites.
Design system to minimize <y> subject to a constraint on the x\_i.
Minimize cost:

C = \sum\_{i=1}^{N\_{sites}} Pr(y\_i) y\_i

Subject to \sum\_{i=1}^{N\_{sites}} x\_i = constant.

1. Cost: Expected size of fire:

C\_{fire} \propto \sum\_{i=1}^{N\_{sites}} p\_i a\_i.

a\_i = area of ith site's region, and p\_i = avg. prob. of fire at ith site over some time frame.

2. Constraint: building and maintaining firewalls. Per unit area, and over same time frame:

C\_{firewalls} \propto \sum\_{i=1}^{N\_{sites}} a\_i^{1/2} a\_i^{-1}.

We are assuming isometry.

In d dimensions, 1/2 is replaced by (d-1)/d

3. Insert question from assignment 7 to find:

Pr(a\_i) \propto a\_i^{-\gamma}

## Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x})p(\vec{x})d\vec{x}$$

where  $C$  is some cost to be evaluated at each point in space  $\vec{x}$  (e.g.,  $V(\vec{x})^\alpha$ ), and  $p(\vec{x})$  is the probability an Ewok jabs position  $\vec{x}$  with a sharpened stick (or equivalent).

2. Constraint:

$$\int R(\vec{x})d\vec{x} = c$$

where  $c$  is a constant.

- Claim/observation is that typically [4]

$$V(\vec{x}) \sim R^{-\beta}(\vec{x})$$

- For spatial systems with barriers:  $\beta = d$ .

## SOC theory

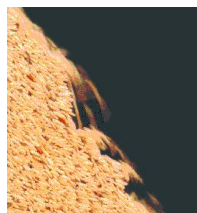
### SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at 'critical states';
- Analogy: Ising model with temperature somehow self-tuning;
- Power-law distributions of sizes and frequencies arise 'for free';
- Introduced in 1987 by Bak, Tang, and Wiesenfeld [3, 2, 8];  
"Self-organized criticality - an explanation of 1/f noise" (PRL, 1987);
- Problem:** Critical state is a very specific point;
- Self-tuning not always possible;
- Much criticism and arguing...



"How Nature Works: the Science of Self-Organized Criticality" [a](#) [a](#)  
by Per Bak (1997). [2]

### Avalanches of Sand and Rice ...



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"Complexity and Robustness" [a](#)  
Carlson and Doyle,  
Proc. Natl. Acad. Sci., **99**, 2538–2545,  
2002. [6]

### HOT versus SOC

- Both produce power laws
- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
- SOC systems have one special density
- HOT systems produce specialized structures
- SOC systems produce generic structures

### HOT theory—Summary of designed tolerance [6]

Table 1. Characteristics of SOC, HOT, and data

Property	SOC	HOT and Data
1 Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2 Robustness	Generic	Robust, yet fragile
3 Density and yield	Low	High
4 Max event size	Infinitesimal	Large
5 Large event shape	Fractal	Compact
6 Mechanism for power laws	Critical internal fluctuations	Robust performance
7 Exponent $\alpha$	Small	Large
8 $\alpha$ vs. dimension $d$	$\alpha \approx (d-1)/10$	$\alpha \approx 1/d$
9 DDOFs	Small (1)	Large ( $\infty$ )
10 Increase model resolution	No change	New structures, new sensitivities
11 Response to forcing	Homogeneous	Variable

### COLD forests

#### Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]
- Weight cost of large losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated

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## Cutoffs

### Observed:

- Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where  $x_c$  is the approximate cutoff scale.

- May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-a x^{-\gamma+1}}$$

## Robustness

### We'll return to this later on:

- Network robustness.
- Albert et al., Nature, 2000:  
"Error and attack tolerance of complex networks" [1]
- General contagion processes acting on complex networks. [13, 12]
- Similar robust-yet-fragile stories ...

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