P o C S What's The Story?

Principles of Complex Systems, Vol. 1, CSYS/MATH 300
University of Vermont, Fall 2020
Assignment 11 • code name: Epidemiology

Due: Friday, November 20, by 4:59 pm, 2020.
Relevant clips, episodes, and slides are listed on the assignment's page:
http://www.uvm.edu/pdodds/teaching/courses/2020-08UVM-300/assignments/11/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Assistant Deliverator: Michael Arnold (contact through Teams)
Office: The Ether
Office hours: Tuesdays, 12 to 12:50 pm; Wednesdays, 1:15 pm to 2:05 pm; Thursdays, 12 to 12:50 pm; all scheduled on Teams
Course website: http://www.uvm.edu/pdodds/teaching/courses/2020-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The Deliverator uses Matlab.

Graduate students are requested to use LaTeX (or related TeX variant). If you are new to LaTeX, please endeavor to submit at least n questions per assignment in LaTeX, where n is the assignment number.

Assignment submission: Via Blackboard.

Please submit your project's current draft in pdf format via Blackboard by the same time specified for this assignment. For teams, please list all team member names clearly at the start.

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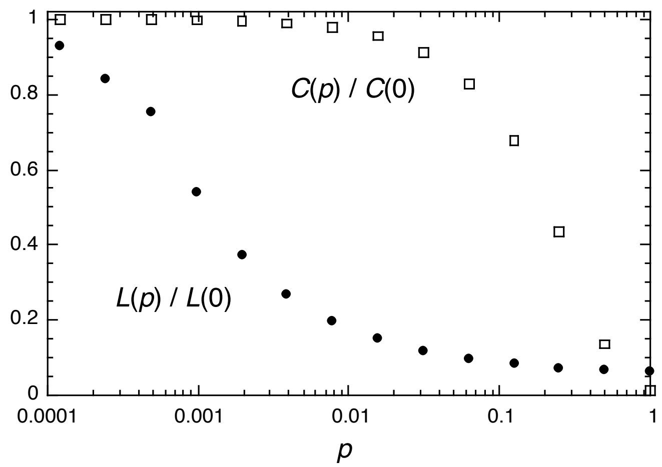
1. Simulate the small-world model and reproduce Fig. 2 from the 1998 Watts-Strogatz paper showing how clustering and average shortest path behave with rewiring probability p [1].

   Please find and use any suitable code online, and feel free to share with each other via Slack.

   Use $N = 1000$ nodes and $k = 10$ for average degree, and vary p from 0.0001 to 1, evenly spaced on a logarithmic scale (there are only 14 values used in the paper).

   Here's the figure you're aiming for:



2. (3 + 3)

   Determine the clustering coefficient for toy model small-world networks [1] as a function of the rewiring probability p. Find C_1, the average local clustering coefficient:

$$C_1(p) = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i = \frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2}$$

   where N is the number of nodes, $a_{ij} = 1$ if nodes i and j are connected, and $\mathcal{N}_i$ indicates the neighborhood of i.

   As per the original model, assume a ring network with each node connected to a fixed, even number m local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

   Start by finding $C_1(0)$ and argue for a $(1-p)^3$ correction factor to find an approximation of $C_1(p)$.

   Hint 1: you can think of finding C_1 as averaging over the possibilities for a single node.

   Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at m. In other words, take the average degree of individuals as the degree of a randomly selected individual.

   For what value of p is $C_1(p)/C_1(0) \simeq 1/2$?

   Does this seem reasonable given your simulation?

   (3 points for set up, 3 for solving.)

References

[1] D. J. Watts and S. J. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998. pdf