1. Given $N$ labelled nodes and allowing for all possible number of edges $m$, what’s the total number of undirected, unweighted networks we can construct? How does this number scale with $N$?

2. Given $N$ labelled nodes and a variable number of $m$ edges, for what value of $m$ do we obtain the largest diversity of networks? And for this $m$, how does the number of networks scale with $N$?

3. We’ve seen that large random networks have essentially no clustering, meaning that locally, random networks are pure branching networks. Nevertheless, a finite, non-zero number of triangles will be present.

   For pure random networks, with connection probability $p = \langle k \rangle / (N - 1)$, what is the expected total number of triangles as $N \to \infty$?

4. Repeat the preceding calculation for cycles of length 4 and 5 (triangles are cycles of length 3).

5. We’ve figured out in class that for large enough $N$ (and $\langle k \rangle$ fixed), a random network always has a Poisson degree distribution:

   $$P(k; \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$
where \( \lambda = \langle k \rangle \). And as we’ve discussed, we don’t find these networks in the real world (they don’t arise due to simple mechanisms). Let’s investigate this oddness a little further.

Compute the expected size of the largest degree in an infinite random network given \( \langle k \rangle \) and as a function of increasing sample size \( N \). In other words, in selecting (with replacement) \( N \) degrees from a pure Poisson distribution with mean \( \langle k \rangle \), what’s the expected minimum value of the largest degree \( \min k_{\text{max}} \)?

A good way to compute \( k_{\text{max}} \) is to equate it to the value for which we expect \( 1/N \) of our random selections to exceed. (We had a question in 300 along these lines for power-law size distributions.)

**Hint**—Of course we’ll be using Stirling’s Approximation.

Direct link: [http://www.youtube.com/v/uK5yakuX59M?rel=0](http://www.youtube.com/v/uK5yakuX59M?rel=0)

6. Show that the second moment of the Poisson distribution is

\[
\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.
\]

and hence that the variance is \( \sigma^2 = \langle k \rangle \).