The Amusing Law of Benford

Benford’s law

References

What’s the Story?

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Outline

Benford’s Law

References
Benford’s Law (تانغ) — The Law of First Digits

\[ P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right) \]

for certain sets of ‘naturally’ occurring numbers in base \(b\)

- Around 30.1% of first digits are ‘1’, compared to only 4.6% for ‘9’.
- First observed by Simon Newcomb\(^2\) in 1881
  “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered in 1938 by Frank Benford (تانغ).
- Newcomb almost always noted but Benford gets the stamp, according to Stigler’s Law of Eponymy. (تانغ)
Benford’s Law—The Law of First Digits

Observed for

- Fundamental constants (electron mass, charge, etc.)
- Utility bills
- Numbers on tax returns (ha!)
- Death rates
- Street addresses
- Numbers in newspapers

- Cited as evidence of fraud in the 2009 Iranian elections.
Benford’s Law—The Law of First Digits

Real data:

Benford’s Law—The Law of First Digits

Physical constants of the universe:

Taken from here ( ).
Benford’s Law—The Law of First Digits

Population of countries:

Taken from [here](#).
Essential story

$P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right)$

$\propto \log_b \left( \frac{d + 1}{d} \right)$

$\propto \log_b (d + 1) - \log_b (d)$

➤ Observe this distribution if numbers are distributed uniformly in log-space:

$P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx$

➤ Power law distributions at work again...

➤ Extreme case of $\gamma \simeq 1$. 
Benford’s law

Taken from here (link).
References I

[1] T. P. Hill.  
The first-digit phenomenon.  

Note on the frequency of use of the different digits in natural numbers.  

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