1. (3 + 3):

Consider a modified version of the Barabási-Albert (BA) model [1] where two possible mechanisms are now in play. As in the original model, start with $m_0$ nodes at time $t = 0$. Let’s make these initial guys connected such that each has degree 1. The two mechanisms are:

M1: With probability $p$, a new node of degree 1 is added to the network. At time $t + 1$, a node connects to an existing node $j$ with probability

$$P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i}$$

(1)

where $k_j$ is the degree of node $j$ and $N(t)$ is the number of nodes in the system at time $t$.

M2: With probability $q = 1 - p$, a randomly chosen node adds a new edge, connecting to node $j$ with the same preferential attachment probability as above.

Note that in the limit $q = 0$, we retrieve the original BA model (with the difference that we are adding one link at a time rather than $m$ here).

In the long time limit $t \rightarrow \infty$, what is the expected form of the degree distribution $P_k$?
Do we move out of the original model’s universality class?
Different analytic approaches are possible including a modification of the BA paper, or a Simon-like one (see also Krapivsky and Redner [2]).
(3 points for set up, 3 for solving.)

References