Voting, Success, and Superstars

Principles of Complex Systems
CSYS/MATH 300, Spring, 2013 | #SpringPoCS2013

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Outline

Winning: it's not for everyone
Superstars
Musiclab

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Winning: it's not for everyone

Superstars
Where do superstars come from?


Examples:
- Full-time Comedians (∼ 200)
- Soloists in Classical Music
- Economic Textbooks (the usual myopic example)
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- Soloists in Classical Music
- Economic Textbooks (the usual myopic example)
- Highly skewed distributions again...
Rosen’s theory:

- Individual quality $q$ maps to reward $R(q)$
- $R(q)$ is ‘convex’ ($d^2 R/dq^2 > 0$)
- Two reasons:
  1. Imperfect substitution:
  2. Technology:

- Joint consumption versus public good
- No social element—success follows ‘inherent quality’
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- Assumes extreme case of equal ‘inherent quality’
- Argues desire for coordination in knowledge and culture leads to differential success
- Success can be purely a social construction
- (How can we measure ‘inherent quality’?)
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Evidence from the web suggestions (Huberman et al.)

1. Easy decisions (yes/no) lead to bandwagoning
   - e.g. jyte.com

2. More costly evaluations lead to oppositional votes
   - e.g. amazon.com

   **Self-selection**: Costly voting may lower incentives for those who agree with the current assessment and increase incentives for those who disagree.
Score-based voting versus rank-based voting:

- Balinski and Laraki [2]
  “A theory of measuring, electing, and ranking”
Laureti et al. (2004): “Aggregating partial, local evaluations to achieve global ranking” [4]

- Model: participants rank \( n \) objects based on underlying quality \( q \)
- Assume evaluation of object \( i \) is a random variable with mean \( q_i \)
- Choose objects based on votes:

\[
p_i(t) \propto v_i(t)^\alpha \text{ or } p_i(t) \propto q_i v_i(t)^\alpha.
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- If \( \alpha < 1 \), correct quality ordering is uncovered
- If \( \alpha > 1 \), some objects are never evaluated and mistakes are made...
- Related to Adler’s approach
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Chase et al. (2002): “Individual differences versus social dynamics in the formation of animal dominance hierarchies” [3]

- The aggressive female Metriaclima zebra:

- Pecking orders for fish...
Dominance hierarchies

Fish forget—changing of dominance hierarchies:

- 22 observations: about 3/4 of the time, hierarchy changed
Dominance hierarchies

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## Dominance hierarchies

### Methods of Forming Hierarchies

<table>
<thead>
<tr>
<th>Size of set</th>
<th>Group assembly</th>
<th>Round-robin competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>A → B → C → D</td>
<td>A → B → C → D</td>
</tr>
<tr>
<td></td>
<td>(23) n=25</td>
<td>(9) n=16</td>
</tr>
<tr>
<td>5</td>
<td>A → B</td>
<td>A → B</td>
</tr>
<tr>
<td></td>
<td>(10) n=11</td>
<td>(1) n=12</td>
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- Group versus isolated interactions produce different hierarchies

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Music Lab Experiment

48 songs
30,000 participants

multiple ‘worlds’
Inter-world variability

- How probable is the world?
- Can we estimate variability?
- Superstars dominate but are unpredictable. Why?
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Experiment 1

Experiments 2–4

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Variability in final rank.
Music Lab Experiment

Variability in final number of downloads.
Inequality as measured by Gini coefficient:

\[ G = \frac{1}{(2N_s - 1)} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} |m_i - m_j| \]
Unpredictability

\[ U = \frac{1}{N_s \binom{N_w}{2}} \sum_{i=1}^{N_s} \sum_{j=1}^{N_w} \sum_{k=j+1}^{N_w} |m_{i,j} - m_{i,k}| \]
Music Lab Experiment

Sensible result:
▶ Stronger social signal leads to greater following and greater inequality.

Peculiar result:
▶ Stronger social signal leads to greater unpredictability.

Very peculiar observation:
▶ The most unequal distributions would suggest the greatest variation in underlying ‘quality.’
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Music Lab Experiment—Sneakiness

- Inversion of download count
- The pretend rich get richer ...
- ... but at a slower rate
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References

Stardom and talent.

A theory of measuring, electing, and ranking.

[3] I. D. Chase, C. Tovey, D. Spangler-Martin, and M. Manfredonia.
Individual differences versus social dynamics in the formation of animal dominance hierarchies.
References II


References III