Benford’s Law: \((\text{있습니다})\)

\[
P(\text{first digit} = d) \propto \log_b \left(1 + \frac{1}{d}\right)
\]

for certain sets of ‘naturally’ occurring numbers in base \(b\)

- Around 30.1% of first digits are ‘1’, compared to only 4.6% for ‘9’.
- First observed by Simon Newcomb\(^\text{[2]}\) in 1881 “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered in 1938 by Frank Benford (있습니다).
- Newcomb almost always noted but Benford gets the stamp.
The law of first digits

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[^2]: Referenced in: [Benford’s Law](https://en.wikipedia.org/wiki/Benford%27s_law)
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- Utility bills
- Numbers on tax returns (ha!)
- Death rates
- Street addresses
- Numbers in newspapers

- Cited as evidence of fraud in the 2009 Iranian elections.
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Real data:

Benford’s Law

Physical constants of the universe:

Taken from here.

References
Benford’s Law

Population of countries:

Taken from [here](#).
Essential story

- \[ P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right) \]

- Observe this distribution if numbers are distributed uniformly in log-space:
  \[ P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx \]

- Power law distributions at work again...
- Extreme case of \( \gamma \approx 1 \).
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References
