1. Consider a modified version of the Barabási-Albert (BA) model [2] where two possible mechanisms are now in play. As in the original model, start with $m_0$ nodes at time $t = 0$. Let’s make these initial guys connected such that each has degree 1. The two mechanisms are:

M1: With probability $p$, a new node of degree 1 is added to the network. At time $t + 1$, a node connects to an existing node $j$ with probability

$$ P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i} $$

where $k_j$ is the degree of node $j$ and $N(t)$ is the number of nodes in the system at time $t$.

M2: With probability $q = 1 - p$, a randomly chosen node adds a new edge, connecting to node $j$ with the same preferential attachment probability as above.

Note that in the limit $q = 0$, we retrieve the original BA model (with the difference that we are adding one link at a time rather than $m$ here).

In the long time limit $t \rightarrow \infty$, what is the expected form of the degree distribution $P_k$?

Do we move out of the original model’s universality class?

(3 points for set up, 3 for solving.)
2. Determine the clustering coefficient for toy model small-world networks [3] as a function of the rewiring probability $p$. Find $C_1$, the average local clustering coefficient:

$$C_1(p) = \left\langle \sum_{j_1,j_2 \in N_i} a_{j_1,j_2} \right\rangle_i = \frac{1}{N} \sum_{i=1}^{N} \sum_{j_1,j_2 \in N_i} a_{j_1,j_2} \frac{k_i(k_i - 1)}{2}$$

where $N$ is the number of nodes, $a_{ij} = 1$ if nodes $i$ and $j$ are connected, and $N_i$ indicates the neighborhood of $i$.

As per the original model, assume a ring network with each node connected to a fixed, even number $m$ local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

Start by finding $C_1(0)$ and argue for a $(1 - p)^3$ correction factor to find an approximation of $C_1(p)$.

Hint 1: you can think of finding $C_1$ as averaging over the possibilities for a single node.

Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at $m$. In other words, take the average degree of individuals as the degree of a randomly selected individual.

For what value of $p$ is $C_1 \simeq 1/2$?

(3 points for set up, 3 for solving.)

3. (Optional)

“Any good idea can be stated in fifty words or less.”—Stanislaw Ulam

Read through Anderson’s seminal paper “More is different” [1] and generate three descriptions of complexification with exactly the following lengths:

(a) Three words,
(b) Six words,
(c) and Twelve words.

Things have sped up since Ulam made his claim. All three may contain one or more sentences.

References


At the very least, Ulam’s claim is self-consistent.