Class Admin

► Office hours:
  ► 1:00 pm to 3:00 pm, Wednesday;
  Farrell Hall, second floor, Trinity Campus.
  ► Appointments by email (peter.dodds@uvm.edu).

► Course outline
► Projects
► Assignments (about 8)
► Assignment 1 appears today and involves:
  ► dolphins
  ► a Karate club
  ► political blogs
  ► a worm’s brain
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  1. Presentation,
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- Presentation versions are navigable and hyperlinks are clickable.

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Bonus materials:

Textbooks:

- Mark Newman (Physics, Michigan)
  “Networks: An Introduction” (荐)

- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  “Networks, Crowds, and Markets: Reasoning About a Highly Connected World” (荐)
Bonus materials:

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [5]
  Times cited: 1,028 (as of June 7, 2010)

- M. Newman
  "The structure and function of complex networks" [16]
  Times cited: 2,559 (as of June 7, 2010)

- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)
Basic definitions:

Complex: (Latin = with + fold/weave (com + plex))

Adjective

► Made up of multiple parts; intricate or detailed.
► Not simple or straightforward.
Basic definitions: Complex System—Some ingredients:

- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence—‘More is Different’[^2]
- Many phenomena can be complex: social, technical, informational, geophysical, meteorological, fluidic, ...
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net•work

noun

1 an arrangement of intersecting horizontal and vertical lines.
   • a complex system of roads, railroads, or other transportation routes: a network of railroads.

2 a group or system of interconnected people or things: a trade network.
   • a group of people who exchange information, contacts, and experience for professional or social purposes: a support network.
   • a group of broadcasting stations that connect for the simultaneous broadcast of a program: the introduction of a second TV network | [as adj.] network television.
   • a number of interconnected computers, machines, or operations: specialized computers that manage multiple outside connections to a network | a local cellular phone network.
   • a system of connected electrical conductors.

verb [trans.]
connect as or operate with a network: the stock exchanges have proven to be resourceful in networking these deals.
• link (machines, esp. computers) to operate interactively: [as adj.] (networked) networked workstations.
• [intrans. ] [often as n. ] (networking) interact with other people to exchange information and develop contacts, esp. to further one's career: the skills of networking, bargaining, and negotiation.
network
noun
1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
2 *a network of lanes* MAZE, labyrinth, warren, tangle.
3 *a network of friends* SYSTEM, complex, nexus, web, webwork.
Ancestry:

From Keith Briggs’s excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?

Ancestry:

First known use: Geneva Bible, 1560
‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

From the OED via Briggs:
► 1658—: reticulate structures in animals
► 1839—: rivers and canals
► 1869—: railways
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Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appears to have long meant purposeful action.

- ‘Network’ = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.
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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
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- Mindboggling amount of work published on complex networks since 1998...
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Popularity (according to ISI)

“Collective dynamics of ‘small-world’ networks” [23]
► Watts and Strogatz
► ≈ 4677 citations (as of January 18, 2011)
► Over 1100 citations in 2008 alone.

“Emergence of scaling in random networks” [3]
► Barabási and Albert
  Science, 1999
► ≈ 5270 citations (as of January 18, 2011)
► Over 1100 citations in 2008 alone.
Popularity according to books:

The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell

Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan
Popularity according to books:

Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási

Six Degrees: The Science of a Connected Age—Duncan Watts
Numerous others:

- Complex Social Networks—F. Vega-Redondo [20]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [17]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [10]
More observations

- But surely **networks aren’t new**...
- Graph theory is well established...
- Study of social networks started in the 1930’s...
- So why all this ‘new’ research on networks?
- **Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- Real networks occupy a tiny, low entropy part of all network space and require specific attention.
- A worthy goal: establish mechanistic explanations.
- What kinds of dynamics lead to these real networks?

* If this is upsetting, maybe string theory is for you...
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- **Web-scale** data sets can be overly *exciting*.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- “The Unreasonable Effectiveness of Data,” Halevy et al. [12]
- c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences” [24]

But:

- For scientists, description is only part of the battle.
- We still need to understand.
More observations

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Super basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...
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- e.g., people, forks in rivers, proteins, webpages, organisms,...
Basic definitions:

**Links** = Connections between nodes

- **links**
  - may be real and fixed (rivers),
  - real and dynamic (airline routes),
  - abstract with physical impact (hyperlinks),
  - or purely abstract (semantic connections between concepts).

- **Links** may be directed or undirected.
- **Links** may be binary or weighted.
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- Notation: Node $i$’s degree = $k_i$.
- $k_i = 0, 1, 2, \ldots$.
- Notation: the average degree of a network = $\langle k \rangle$.

For undirected networks, connection between number of edges $m$ and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

For directed networks,

$$\langle k_{\text{out}} \rangle = \langle k_{\text{in}} \rangle = \frac{m}{N}$$

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Adjacency matrix:

- We represent a graph or network by a matrix $A$ with link weight $a_{ij}$ for nodes $i$ and $j$ in entry $(i, j)$.

  - e.g.,

    $$ A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} $$

- (n.b., for numerical work, we always use sparse matrices.)
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What passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
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- Complex networks can be social, economic, natural, informational, abstract, ...
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Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

- Distribution (branching) versus redistribution (cyclical)
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Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
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- Call networks (AT&T)
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Examples of Complex Networks

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Modelling Complex Networks

Nutshell

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datamining.typepad.com
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More examples can be found at datamining.typepad.com
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[Link to data mining example](datamining.typepad.com)
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datamining.typepad.com (田)
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Interaction networks: social networks

► Snogging
► Friendships
► Acquaintances
► Boards and directors
► Organizations

► twitter.com (⊞)
► facebook.com (⊞)

► ‘Remotely sensed’ by: tweets (open), instant messaging, Facebook posts, emails, phone logs (*cough*)

(Bearman et al., 2004)
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Relational networks

- Consumer purchases
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: delicious, flickr
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Bollen et al. [6]; a higher resolution figure is [here](#)
A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.

- And even when renderings somehow look good:

- We need to extract digestible, meaningful aspects.
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  - number of nodes $N = 500$
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Properties

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

* Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of...
1. degree distribution $P_k$

- $P_k$ is the probability that a randomly selected node has degree $k$
- $k = \text{node degree} = \text{number of connections}$
- ex 1: Erdős–Rényi random networks:

$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

- Distribution is Poisson
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► ex 2: “Scale-free” networks: $P_k \propto k^{-\gamma} \Rightarrow \text{‘hubs’}$

► link cost controls skew
► hubs may facilitate or impede contagion
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Note:

- Erdős-Rényi random networks are a *mathematical construct*.
- ‘Scale-free’ networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.
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2. Assortativity/3. Homophily:

- Social networks: Homophily (знак) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network: similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.
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Local socialness:

4. Clustering:

- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz\(^{[23]}\)
     \[
     C_1 = \left\langle \frac{\sum_{j \in \mathcal{N}_i} a_{ij}a_{jk}}{k_i(k_i - 1)/2} \right\rangle_i
     \]
  2. Newman\(^{[16]}\)
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     C_2 = \frac{3 \times \text{#triangles}}{\text{#triples}}
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First clustering measure:

Example network:

▶ $C_1$ is the average fraction of pairs of neighbors who are connected.

▶ Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1,j_2 \in N_i} a_{j_1,j_2}}{k_i(k_i-1)/2}$$

where $k_i$ is node $i$'s degree, and $N_i$ is the set of $i$'s neighbors.

▶ Averaging over all nodes, we have:

$$C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1,j_2 \in N_i} a_{j_1,j_2}}{k_i(k_i-1)/2}$$
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Calculation of $C_1$:

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  \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}
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  where \( k_i \) is node \( i \)'s degree, and \( \mathcal{N}_i \) is the set of \( i \)'s neighbors.
- Averaging over all nodes, we have:
  \[
  C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}
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First clustering measure:

Example network:

Calculation of $C_1$:

- $C_1$ is the **average fraction of pairs of neighbors who are connected**.
- Fraction of pairs of neighbors who are connected is

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Triples and triangles

Example network:

- **Triangles:**
  - Nodes $i_1$, $i_2$, and $i_3$ form a triangle if each pair of nodes is connected.

- **Triples:**
  - The definition $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ measures the fraction of closed triples.
  - The ‘3’ appears because for each triangle, we have 3 closed triples.
  - Social Network Analysis (SNA): fraction of transitive triples.

- Nodes $i_1$, $i_2$, and $i_3$ form a triple around $i_1$ if $i_1$ is connected to $i_2$ and $i_3$.
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- For sparse networks, $C_1$ tends to discount highly connected nodes.
- $C_2$ is a useful and often preferred variant.
- In general, $C_1 \neq C_2$.
- $C_1$ is a global average of a local ratio.
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- small, recurring functional subnetworks
- e.g., Feed Forward Loop:

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![Feed Forward Loop Diagram]

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6. modularity and structure/community detection:

Clauset et al., 2006\textsuperscript{[9]}: NCAA football
7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
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8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: \( R_n = \frac{N_\omega}{N_{\omega+1}} \)
  - Segment length: \( R_l = \frac{\langle l_{\omega+1} \rangle}{\langle l_\omega \rangle} \)
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(a) shortest path length $d_{ij}$:
- Fewest number of steps between nodes $i$ and $j$.
- (Also called the chemical distance between $i$ and $j$.)

(b) average path length $\langle d_{ij} \rangle$:
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- Closeness handles disconnected networks ($d_{ij} = \infty$)
- $d_{\text{cl}} = \infty$ only when all nodes are isolated.
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- Many such measures of a node's 'importance.'
- **ex 1:** Degree centrality: $k_i$.
- **ex 2:** Node $i$'s betweenness
  - fraction of shortest paths that pass through $i$.
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1. generalized random networks (touched on in 300)
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- Generative model.
- Preferential attachment model with growth:
  - $P[\text{attachment to node } i] \propto k_i^\alpha$.
  - Produces $P_k \sim k^{-\gamma}$ when $\alpha = 1$.
- Trickiness: other models generate skewed degree distributions.

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$\langle k \rangle = 1.8$
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5. generalized affiliation networks

Bipartite affiliation networks: boards and directors, movies and actors.
5. generalized affiliation networks
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- Blau & Schwartz [4], Simmel [19], Breiger [8], Watts et al. [22]

Models

Overview

Class admin

Basic definitions

Popularity

Examples of Complex Networks

Properties of Complex Networks

Modelling Complex Networks

Nutshell

References
Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
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- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).
Overview Key Points (cont.):

▶ Obvious connections with the vast extant field of graph theory.
▶ But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
▶ Two main areas of focus:
  1. Description: Characterizing very large networks
  2. Explanation: Micro story ⇒ Macro features
▶ Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure, ...
▶ Still much work to be done, especially with respect to dynamics... exciting!
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