Overview of Complex Networks

Complex Networks
CSYS/MATH 303, Spring, 2011

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Outline

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Class Admin

- Office hours:
  - 1:00 pm to 3:00 pm, Wednesday: Farrell Hall, second floor, Trinity Campus.
  - Appointments by email (peter.dodds@uvm.edu).
- Course outline
- Projects
- Assignments (about 8)
- Assignment 1 appears today and involves:
  - dolphins
  - a Karate club
  - political blogs
  - a worm’s brain
  - the Internet
  - jazz musicians

Exciting details regarding these slides:

- Three versions (all in pdf):
  1. Presentation,
  2. Flat Presentation,
  3. Handout (2x2).
- Presentation versions are navigable and hyperlinks are clickable.
- Web links look like this (⊞).
- References in slides link to full citation at end. [2]
- Citations contain links to papers in pdf (if available).
- Brought to you by a troubling concoction of LaTeX, Beamer, and perl.

Bonus materials:

Textbooks:
- Mark Newman (Physics, Michigan)
  “Networks: An Introduction” [3]
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  “Networks, Crowds, and Markets: Reasoning About a Highly Connected World” [3]

Review articles:
- S. Boccaletti et al.
  “Complex networks: structure and dynamics” [5]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  “The structure and function of complex networks” [16]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  “Statistical mechanics of complex networks” [1]
  Times cited: 3,995 (as of June 7, 2010)
Basic definitions:

Complex: (Latin = with + fold/weave (com + plex))

Adjective
- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions:

Complex System—Some ingredients:
- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Emergence—More is Different
- Many phenomena can be complex: social, technical, informational, geophysical, meteorological, fluidic, ...

network | [network]
noun
1 an arrangement of intersecting horizontal and vertical lines.
- a complex system of roads, railroads, or other transportation routes: a network of railroads.
- a group of people who exchange information, contacts, and experience for professional or social purposes: a social network.
- a group of broadcasting stations that connect for the simultaneous broadcast of a program: the introduction of a second TV network.
- a number of interconnected computers, machines, or operations: specialized computers that manage multiple outside connections to a network.
- a local cellular phone network.
- a system of connected electrical conductors.

verb
connect as or operate with a network: the stock exchanges have proven to be resourceful in networking these deals.
- link (machines, esp. computers) to operate interactively: networked; networked applications.

Ancestry:

From Keith Briggs’s excellent etymological investigation:

- Opus reticulatum:
- A Latin origin?

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Thesaurus deliciousness:

network
- a network of arteries, WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- a network of lanes, MAZE, labyrinth, warren, tangle.
- a network of friends, SYSTEM, complex, nexus, web, webwork.
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Ancestry:

First known use: Geneva Bible, 1560
‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

From the OED via Briggs:
» 1658–: reticulate structures in animals
» 1839–: rivers and canals
» 1869–: railways
» 1883–: distribution network of electrical cables
» 1914–: wireless broadcasting networks

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Ancestry:

Net and Work are venerable old words:
» ‘Net’ first used to mean spider web (King Ælfréd, 888).
» ‘Work’ appears to have long meant purposeful action.

» ‘Network’ = something built based on the idea of natural, flexible lattice or web.
» c.f., ironwork, stonework, fretwork.

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Key Observation:

» Many complex systems can be viewed as complex networks of physical or abstract interactions.
» Opens door to mathematical and numerical analysis.
» Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
» Mindboggling amount of work published on complex networks since 1998...
» ... largely due to your typical theoretical physicist:

» Piranha physicus
» Hunt in packs.
» Feast on new and interesting ideas (see chaos, cellular automata, ...)

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Popularity (according to ISI)

“Collective dynamics of ‘small-world’ networks”[23]
» Watts and Strogatz
» ≈ 4677 citations (as of January 18, 2011)
» Over 1100 citations in 2008 alone.

“Emergence of scaling in random networks”[3]
» Barabási and Albert
Science, 1999
» ≈ 5270 citations (as of January 18, 2011)
» Over 1100 citations in 2008 alone.

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Popularity according to books:


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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Popularity according to books:

Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási

Six Degrees: The Science of a Connected Age—Duncan Watts[21]

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Numerous others:
- Complex Social Networks—F. Vega-Redondo
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes

More observations:
- But surely networks aren’t new...
- Graph theory is well established...
- Study of social networks started in the 1930’s...
- So why all this ‘new’ research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alias) our theories with a much more measurable reality.*
- Real networks occupy a tiny, low entropy part of all network space and require specific attention.
- A worthy goal: establish mechanistic explanations.
- What kinds of dynamics lead to these real networks?
  *If this is upsetting, maybe string theory is for you...

More observations:
- Web-scale data sets can be overly exciting.

Witness:
- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- “The Unreasonable Effectiveness of Data,” Halevy et al.
- c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences"

But:
- For scientists, description is only part of the battle.
- We still need to understand.

Nodes = A collection of entities which have properties that are somehow related to each other
- e.g., people, forks in rivers, proteins, webpages, organisms,...

**Super basic definitions**

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**Nodes**
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**Basic definitions:**

**Links** = Connections between nodes
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**Basic definitions:**

**Node degree** = Number of links per node
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Basic definitions:

Adjacency matrix:

- We represent a graph or network by a matrix \( A \) with link weight \( a_{ij} \) for nodes \( i \) and \( j \) in entry \((i, j)\).
- e.g.,
  \[
  A = \begin{pmatrix}
  0 & 1 & 1 & 0 \\
  0 & 1 & 0 & 1 \\
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 1 \\
  0 & 1 & 0 & 1 \\
  \end{pmatrix}
  \]
  (n.b., for numerical work, we always use sparse matrices.)

Examples

What passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (W)
- Airline networks
- Call networks (AT&T)
- The Media

Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- twitter.com (open), instant messaging, Facebook posts, emails, phone logs (*cough*).

Examples

Relational networks

- Consumer purchases (Wal-Mart: \( 2.5 \times 10^{15} \) bytes)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: delicious (open), flickr (open)
A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.
- Typical hairball
  - number of nodes $N = 500$
  - number of edges $m = 1000$
  - average degree $\langle k \rangle = 4$
- And even when renderings somehow look good: “That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way” said Ponder [Stibbons] — Making Money, T. Pratchett.
- We need to extract digestible, meaningful aspects.

**Properties**

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

*Plus coevolution of network structure and processes on networks.

- Degree distribution is the elephant in the room that we are now all very aware of...

1. degree distribution $P_k$

- $P_k$ is the probability that a randomly selected node has degree $k$
- $k = \text{node degree} = \text{number of connections}$
- ex 1: Erdős-Rényi random networks:
  $$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$
- Distribution is Poisson

1. degree distribution $P_k$

- ex 2: “Scale-free” networks: $P_k \propto k^{-\gamma} \Rightarrow \text{hubs}$
- link cost controls skew
- hubs may facilitate or impede contagion

Note:

- Erdős-Rényi random networks are a mathematical construct.
- ‘Scale-free’ networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.
Properties

2. Assortativity/3. Homophily:
- Social networks: Homophily (≡) = birds of a feather e.g., degree is standard property for sorting; measure degree-degree correlations.
- Assortative network: similar degree nodes connecting to each other. Often social: company directors, coauthors, actors.
- Disassortative network: high degree nodes connecting to low degree nodes. Often technological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Local socialness:

4. Clustering:
- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz \[ C_1 = \left( \frac{\sum_{i=1}^{n} \sum_{j \in N_i} A_{ij} - n}{k_i(k_i - 1)/2} \right) \]
  2. Newman \[ C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}} \]

First clustering measure:

Example network:

Calculation of \( C_1 \):

\[ C_1 = \frac{\sum_{i=1}^{n} \sum_{j \in N_i} A_{ij} - n}{k_i(k_i - 1)/2} \]

where \( k_i \) is node \( i \)'s degree, and \( N_i \) is the set of \( i \)'s neighbors.

Averaging over all nodes, we have:

\[ C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j \in N_i} A_{ij}}{k_i(k_i - 1)/2} \]

Triangles and triangles

Example network:

- Nodes \( i_1 \), \( i_2 \), and \( i_3 \) form a triangle if each pair of nodes is connected
- The definition \( C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}} \) measures the fraction of closed triangles
- The '3' appears because for each triangle, we have 3 closed triangles.
- Social Network Analysis (SNA): fraction of transitive triples.

Properties

5. motifs:
- small, recurring functional subnetworks e.g., Feed Forward Loop:

\[
\begin{align*}
\text{FeedForwardLoop} & \\
\text{Shen-Orr, Uri Alon, et al.} [18]
\end{align*}
\]
Properties

6. modularity and structure/community detection:

Clauset et al., 2006[^9]: NCAA football

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9. network distances:

(a) shortest path length \(d_{ij}\):

- Fewest number of steps between nodes \(i\) and \(j\).
- (Also called the chemical distance between \(i\) and \(j\)).

(b) average path length \(\langle d_{ij} \rangle\):

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
- Weighted links can be accommodated.

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7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996[^14]

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8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: \(R_0 = N_0 / N_{0+1}\)
  - Segment length: \(R_s = \langle u_{i+1} \rangle / \langle u_i \rangle\)
  - Area/Volume: \(R_A = \langle a_{i+1} \rangle / \langle a_i \rangle\)

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9. network distances:

- network diameter \(d_{max}\):
  - Maximum shortest path length between any two nodes.
- closeness \(d_{cl} = \left[ \sum_{ij} \frac{d_{ij} - 1}{n^2} \right]^{-1}\):
  - Average ‘distance’ between any two nodes.
  - Closeness handles disconnected networks (\(d_{ij} = \infty\))
  - \(d_{cl} = \infty\) only when all nodes are isolated.
  - Closeness perhaps compresses too much into one number

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10. centrality:

- Many such measures of a node’s ‘importance.’
  - ex 1: Degree centrality: \(k_i\).
  - ex 2: Node \(i\)’s betweenness
    - fraction of shortest paths that pass through \(i\).
  - ex 3: Edge \(\ell\)’s betweenness
    - fraction of shortest paths that travel along \(\ell\).
  - ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg[^13])

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Models

Some important models:

1. generalized random networks (touched on in 300)
2. scale-free networks (⊞) (covered in 300)
3. small-world networks (⊞) (covered in 300)
4. statistical generative models ($p^*$)
5. generalized affiliation networks (partly covered in 300)

1. generalized random networks:
   - Arbitrary degree distribution $P_k$.
   - Wire nodes together randomly.
   - Create ensemble to test deviations from randomness.
   - Interesting, applicable, rich mathematically.
   - We will have fun with these guys...

2. ‘scale-free’ networks:
   - Introduced by Barabasi and Albert[2]
   - Generative model
   - Preferential attachment model with growth:
     $P(attachment \ to \ node \ i) \sim k_i^\alpha$.
   - Produces $P_k \sim k^{-\gamma}$ when $\alpha = 1$.
   - Trickiness: other models generate skewed degree distributions.

3. small-world networks
   - Introduced by Watts and Strogatz[3]
   - Two scales:
     - local regularity (an individual’s friends know each other)
     - global randomness (shortcuts).
   - Shortcuts allow disease to jump
   - Number of infectives increases exponentially in time
   - Facilitates synchronization

Bipartite affiliation networks: boards and directors, movies and actors.
Models

5. generalized affiliation networks

graph

- Blau & Schwartz [4], Simmel [19], Breiger [8], Watts et al. [22]

Nutshell:

Overview Key Points:
- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

References

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