The law of first digits

Benford's Law: \( P(\text{first digit} = d) \propto \log_b (1 + 1/d) \)

- for certain sets of 'naturally' occurring numbers in base \( b \)
- Around 30.1% of first digits are '1', compared to only 4.6% for '9'.
- First observed by Simon Newcomb \(^2\) in 1881
  "Note on the Frequency of Use of the Different Digits in Natural Numbers"
- Independently discovered in 1938 by Frank Benford \(^3\).
- Newcomb almost always noted but Benford gets the stamp.

Benford's Law—The Law of First Digits

Observed for
- Fundamental constants (electron mass, charge, etc.)
- Utility bills
- Numbers on tax returns (ha!)
- Death rates
- Street addresses
- Numbers in newspapers
- Cited as evidence of fraud \( \text{(III)} \) in the 2009 Iranian elections.

References

The Amusing Law of Benford
Principles of Complex Systems
CSYS/MATH 300, Fall, 2010

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References

Outline

Benford's Law

Benford's Law

Real data

Benford's Law

Physical constants of the universe:

Taken from here \( \text{III} \).

\(^1\) T. P. Hill (1998)
\(^2\) Simon Newcomb
\(^3\) Frank Benford
Benford's Law

Population of countries:

![Bar chart showing population distribution.]

Taken from [here](#).

Essential story

\[
P(\text{first digit} = d) \propto \log_b \left( \frac{d + 1}{d} \right) = \log_b (d + 1) - \log_b (d)
\]

- Observe this distribution if numbers are distributed uniformly in log-space:
  \[
P(\ln x) d(\ln x) \propto 1 - d(\ln x) = x^{-1} dx
\]
- Power law distributions at work again...
- Extreme case of \( \gamma \approx 1 \).

Benford's law

![Graph showing Benford's law distribution.]

Taken from [here](#).

References