1. Consider a modified version of the Barabási-Albert (BA) model \cite{[1]} where two possible mechanisms are now in play. As in the original model, start with \(m_0\) nodes at time \(t = 0\). Let’s make these initial guys connected such that each has degree 1. The two mechanisms are:

M1: With probability \(p\), a new node of degree 1 is added to the network. At time \(t + 1\), a node connects to an existing node \(j\) with probability

\[
P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i} \tag{1}
\]

where \(k_j\) is the degree of node \(j\) and \(N(t)\) is the number of nodes in the system at time \(t\).

M2: With probability \(q = 1 - p\), a randomly chosen node adds a new edge, connecting to node \(j\) with the same preferential attachment probability as above.

Note that in the limit \(q = 0\), we retrieve the original BA model (with the difference that we are adding one link at a time rather than \(m\) here).

In the long time limit \(t \to \infty\), what is the expected form of the degree distribution \(P_k\)?

Do we move out of the original model’s universality class?
2. Now take the Barabási-Albert model with an attachment kernel $A_k = k^{1/2}$. Take newly arriving nodes as adding $m$ links ($m = 1$ for the preceding question).

Use the same approach as in class (which is a modified version of the original derivation in [1]), to determine the long-time limiting form of the degree distribution $P_k$.

A catch and a hint: to normalize the attachment kernel at each point in time $t$, we have to divide by the sum of all degrees in the network (as per Eq. [1] above). Recall that for the original model, the sum of all degrees nicely simplified to $2mt + m_0$ (check over this). But now we have the sum of $k_i^{1/2}$, and its form is not obvious. Here’s the help: assume that

$$\sum_{i=1}^{N(t)} k_i^{1/2} = \lambda t$$

where $\lambda$ is to be determined later. In other words, assume that the normalization factor grows linearly with $t$, as it did for the original model. If this is indeed true, then you will be able to justify it once you have found $P_k$.

References