Lecture 1/25—Chapter 2
Linear Algebra
MATH 124, Fall, 2010
Prof. Peter Dodds
Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont

Outlines:
- Outline
- Importance
- Usages
- Key problems
- Three ways of looking...
- Colbert on Equations
- References

Grading breakdown:
1. Assignments (40%)
   - Ten one-week assignments.
   - Lowest assignment score will be dropped.
   - The last assignment cannot be dropped.
   - Each assignment will have a random bonus point question which has nothing to do with linear algebra.

2. Midterm exams (35%)
   - Three 75 minutes tests distributed throughout the course, all of equal weighting.

3. Final exam (24%)
   - ≤ Three hours of joyful celebration.
   - Saturday, December 11, 7:30 am to 10:15 am, 209 Votey

Basics:
- Instructor: Prof. Peter Dodds
- Lecture room and meeting times:
  209 Votey Hall, Tuesday and Thursday, 10:00 am to 11:15 am
- Office: Farrell Hall, second floor, Trinity Campus
- E-mail: peter.dodds@uvm.edu
- Course website: http://www.uvm.edu/~pdodds/teaching/courses/2010-08UVM-124

Paper products:
1. Outline

Papers to read:
1. “The Fundamental Theorem of Linear Algebra”[1]
2. “Too Much Calculus”[2]

Office hours:
- 1:00 pm to 4:00 pm, Wednesday, Farrell Hall, second floor, Trinity Campus

1. Assignment (0%)—Problems assigned online from the textbook. Doing these exercises will be most beneficial and will increase happiness.
2. General attendance (1%)—it is extremely desirable that students attend class, and class presence will be taken into account if a grade is borderline.
How grading works:

Questions are worth 3 points according to the following scale:

- 3 = correct or very nearly so.
- 2 = acceptable but needs some revisions.
- 1 = needs major revisions.
- 0 = way off.

Schedule:

The course will mainly cover chapters 2 through 6 of the textbook. (You should know all about Chapter 1.)

<table>
<thead>
<tr>
<th>Week # (dates)</th>
<th>Tuesday</th>
<th>Thursday</th>
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</thead>
<tbody>
<tr>
<td>1 (8/31, 9/2)</td>
<td>Lecture</td>
<td>Lecture + A1</td>
</tr>
<tr>
<td>2 (9/7, 9/9)</td>
<td>Lecture</td>
<td>Lecture + A2</td>
</tr>
<tr>
<td>3 (9/14, 9/16)</td>
<td>Lecture</td>
<td>Lecture + A3</td>
</tr>
<tr>
<td>4 (9/21, 9/23)</td>
<td>Lecture</td>
<td>Test 1</td>
</tr>
<tr>
<td>5 (9/28, 9/30)</td>
<td>Lecture</td>
<td>Lecture + A4</td>
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<tr>
<td>6 (10/5, 10/7)</td>
<td>Lecture</td>
<td>Lecture + A5</td>
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<tr>
<td>7 (10/12, 10/14)</td>
<td>Lecture</td>
<td>Lecture + A6</td>
</tr>
<tr>
<td>8 (10/19, 10/21)</td>
<td>Lecture</td>
<td>Test 2</td>
</tr>
<tr>
<td>9 (10/26, 10/29)</td>
<td>Lecture</td>
<td>Lecture + A7</td>
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<tr>
<td>10 (11/2, 11/4)</td>
<td>Lecture</td>
<td>Lecture + A8</td>
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<tr>
<td>11 (11/9, 11/11)</td>
<td>Lecture</td>
<td>Lecture + A9</td>
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<tr>
<td>12 (11/16, 11/18)</td>
<td>Lecture</td>
<td>Test 3</td>
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<tr>
<td>13 (11/23, 11/25)</td>
<td>Thanksgiving</td>
<td>Thanksgiving</td>
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<tr>
<td>14 (11/30, 12/2)</td>
<td>Lecture</td>
<td>Lecture + A10</td>
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<tr>
<td>15 (12/7, 12/9)</td>
<td>Lecture</td>
<td>Lecture</td>
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Important dates:

1. Classes run from Monday, August 31 to Wednesday, December 9.
3. Last day to withdraw—Friday, November 6.
4. Reading and exam period—Thursday, December 10 to Friday, December 18.

More stuff:

Do check your zoo account for updates regarding the course.

Academic assistance: Anyone who requires assistance in any way (as per the ACCESS program or due to athletic endeavors), please see or contact me as soon as possible.

Being good people:

1. In class there will be no electronic gadgetry, no cell phones, no beeping, no text messaging, etc. You really just need your brain, some paper, and a writing implement here (okay, and Matlab or similar).
2. Second, I encourage you to email me questions, ideas, comments, etc., about the class but request that you please do so in a respectful fashion.
3. Finally, as in all UVM classes, Academic honesty will be expected and departures will be dealt with appropriately. See [http://www.uvm.edu/cses/](http://www.uvm.edu/cses/) for guidelines.

Late policy: Unless in the case of an emergency (a real one) or if an absence has been predeclared and a make-up version sorted out, assignments that are not turned in on time or tests that are not attended will be given 0%.

Computing: Students are encouraged to use Matlab or something similar to check their work.

Note: for assignment problems, written details of calculations will be required.
Grading:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>A+</td>
<td>97–100</td>
</tr>
<tr>
<td>A</td>
<td>93–96</td>
</tr>
<tr>
<td>A-</td>
<td>90–92</td>
</tr>
<tr>
<td>B+</td>
<td>87–89</td>
</tr>
<tr>
<td>B</td>
<td>83–86</td>
</tr>
<tr>
<td>B-</td>
<td>80–82</td>
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<tr>
<td>C+</td>
<td>77–79</td>
</tr>
<tr>
<td>C</td>
<td>73–76</td>
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<tr>
<td>C-</td>
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<td>D+</td>
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<tr>
<td>D</td>
<td>63–66</td>
</tr>
<tr>
<td>D-</td>
<td>60–62</td>
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</tbody>
</table>

Why are we doing this?

Big deal: Linear Algebra is a body of mathematics that deals with discrete problems.

Many things are discrete:
- Information (0’s & 1’s, letters, words)
- People (sociology)
- Networks (the Web, people again, food webs, ...)
- Sounds (musical notes)

Even more: If real data is continuous, we almost always discretize it (0’s and 1’s)

Matrices as gadgets:

A matrix $A$ transforms a vector $\vec{x}$ into a new vector $\vec{x}'$ through matrix multiplication (whatever that is):

$$\vec{x}' = A \vec{x}$$

We can use matrices to:
- Grow vectors
- Shrink vectors
- Rotate vectors
- Flip vectors
- Do all these things in different directions
- Reveal the true ur-dystopian reality.

Three key problems of Linear Algebra

1. Given a matrix $A$ and a vector $\vec{b}$, find $\vec{x}$ such that

$$A\vec{x} = \vec{b}.$$ 

2. Eigenvalue problem: Given $A$, find $\lambda$ and $\vec{v}$ such that

$$A\vec{v} = \lambda \vec{v}.$$ 

3. Coupled linear differential equations:

$$\frac{dy(t)}{dt} = Ay(t)$$

- Our focus will be largely on #1, partly on #2.
**Major course objective:**

To deeply understand the equation \( A\vec{x} = \vec{b} \), the Fundamental Theorem of Linear Algebra, and the following picture:

![Diagram of Row Space and Column Space](image)

What is going on here? We have 25 lectures to find out...

**Our friend \( A\vec{x} = \vec{b} \):**

What does knowing \( \vec{x} \) give us?

If we can represent reality as a superposition (or combination or sum) of simple elements, we can do many things:

- Compress information
- See how we can alter information (filtering)
- Find a system’s simplest representation
- Find a system’s most important elements
- See how to adjust a system in a principled way

**Three ways to understand \( A\vec{x} = \vec{b} \):**

- **Way 1: The Row Picture**
- **Way 2: The Column Picture**
- **Way 3: The Matrix Picture**

Example:

\[
\begin{align*}
-x_1 + x_2 &= 1 \\
2x_1 + x_2 &= 4
\end{align*}
\]

- Call this a 2 by 2 system of equations.
- 2 equations with 2 unknowns.
- Standard method of simultaneous equations: solve above by adding and subtracting multiples of equations to each other = Row Picture.

**Our new BFF: \( A\vec{x} = \vec{b} \):**

Broadly speaking, \( A\vec{x} = \vec{b} \) translates as follows:

- \( \vec{b} \) represents reality (e.g., music, structure)
- \( A \) contains building blocks (e.g., notes, shapes)
- \( \vec{x} \) specifies how we combine our building blocks to make \( \vec{b} \) (as best we can).

How can we disentangle an orchestra’s sound?

What about pictures, waves, signals, ...?
Three ways to understand $A\vec{x} = \vec{b}$:

The column picture:

See

$$-x_1 + x_2 = 1$$
$$2x_1 + x_2 = 4$$

as

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$ 

General problem

$$x_1 \vec{a}_1 + x_2 \vec{a}_2 = \vec{b}$$

- Column vectors are our ‘building blocks’
- Key idea: try to ‘reach’ $\vec{b}$ by combining (summing) multiples of column vectors $\vec{a}_1$ and $\vec{a}_2$.

Three ways to understand $A\vec{x} = \vec{b}$:

We love the column picture:

- Intuitive.
- Generalizes easily to many dimensions.

Three possible kinds of solution:

1. $\vec{a}_1 \parallel \vec{a}_2$: 1 solution
2. $\vec{a}_1 \parallel \vec{a}_2 \parallel \vec{b}$: No solutions
3. $\vec{a}_1 \parallel \vec{a}_2 \parallel \vec{b}$: infinitely many solutions

(assuming neither $\vec{a}_1$ or $\vec{a}_2$ are 0)

Three ways to understand $A\vec{x} = \vec{b}$:

Difficulties:

- Do we give up if $A\vec{x} = \vec{b}$ has no solution?
- No! We can still find the $\vec{x}$ that gets us as close to $\vec{b}$ as possible.
- Method of approximation—very important!
- We may not have the right building blocks but we can do our best.

The truth about mathematics

The Colbert Report on Math (⊞) (February 7, 2006)
References I

   The fundamental theorem of linear algebra.

   Too much calculus, 2002.
   *SIAM Linear Algebra Activity Group Newsletter*. pdf