Outline

Importance

Usages

Key problems

Three ways of looking...

Colbert on Equations

References
Basics:

- **Instructor:** Prof. Peter Dodds
- **Lecture room and meeting times:**
  209 Votey Hall, Tuesday and Thursday, 10:00 am to 11:15 am
- **Office:** Farrell Hall, second floor, Trinity Campus
- **E-mail:** peter.dodds@uvm.edu
- **Course website:** [http://www.uvm.edu/~pdodds/teaching/courses/2010-08UVM-124](http://www.uvm.edu/~pdodds/teaching/courses/2010-08UVM-124)
Admin:

Paper products:

1. Outline

Papers to read:


Office hours:

- 1:00 pm to 4:00 pm, Wednesday,
  Farrell Hall, second floor, Trinity Campus
Grading breakdown:

1. Assignments (40%)
   - Ten one-week assignments.
   - Lowest assignment score will be dropped.
   - The last assignment cannot be dropped!
   - Each assignment will have a random bonus point question which has nothing to do with linear algebra.

2. Midterm exams (35%)
   - Three 75 minutes tests distributed throughout the course, all of equal weighting.

3. Final exam (24%)
   - Three hours of joyful celebration.
   - Saturday, December 11, 7:30 am to 10:15 am, 209 Votey
Grading breakdown:

1. **Homework (0%)**—Problems assigned online from the textbook. Doing these exercises will be most beneficial and will increase happiness.

2. **General attendance (1%)**—it is extremely desirable that students attend class, and class presence will be taken into account if a grade is borderline.
How grading works:

Questions are worth 3 points according to the following scale:

- 3 = correct or very nearly so.
- 2 = acceptable but needs some revisions.
- 1 = needs major revisions.
- 0 = way off.
The course will mainly cover chapters 2 through 6 of the textbook. (You should know all about Chapter 1.)

<table>
<thead>
<tr>
<th>Week # (dates)</th>
<th>Tuesday</th>
<th>Thursday</th>
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<tbody>
<tr>
<td>1 (8/31, 9/2)</td>
<td>Lecture</td>
<td>Lecture + A1</td>
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<tr>
<td>2 (9/7, 9/9)</td>
<td>Lecture</td>
<td>Lecture + A2</td>
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<tr>
<td>3 (9/14, 9/16)</td>
<td>Lecture</td>
<td>Lecture + A3</td>
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<tr>
<td>4 (9/21, 9/23)</td>
<td>Lecture</td>
<td>Test 1</td>
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<td>5 (9/28, 9/30)</td>
<td>Lecture</td>
<td>Lecture + A4</td>
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<tr>
<td>6 (10/5, 10/7)</td>
<td>Lecture</td>
<td>Lecture + A5</td>
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<tr>
<td>7 (10/12, 10/14)</td>
<td>Lecture</td>
<td>Lecture + A6</td>
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<tr>
<td>8 (10/19, 10/21)</td>
<td>Lecture</td>
<td>Test 2</td>
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<tr>
<td>9 (10/26, 10/29)</td>
<td>Lecture</td>
<td>Lecture + A7</td>
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<td>10 (11/2, 11/4)</td>
<td>Lecture</td>
<td>Lecture + A8</td>
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<tr>
<td>11 (11/9, 11/11)</td>
<td>Lecture</td>
<td>Lecture + A9</td>
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<tr>
<td>12 (11/16, 11/18)</td>
<td>Lecture</td>
<td>Test 3</td>
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<td>13 (11/23, 11/25)</td>
<td>Thanksgiving</td>
<td>Thanksgiving</td>
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<tr>
<td>14 (11/30, 12/2)</td>
<td>Lecture</td>
<td>Lecture + A10</td>
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<tr>
<td>15 (12/7, 12/9)</td>
<td>Lecture</td>
<td>Lecture</td>
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Important dates:

1. Classes run from Monday, August 31 to Wednesday, December 9.
3. Last day to withdraw—Friday, November 6.
4. Reading and exam period—Thursday, December 10 to Friday, December 18.
More stuff:

**Do** check your zoo account for updates regarding the course.

**Academic assistance:** Anyone who requires assistance in any way (as per the ACCESS program or due to athletic endeavors), please see or contact me as soon as possible.
More stuff:

Being good people:

1. In class there will be no electronic gadgetry, no cell phones, no beeping, no text messaging, etc. You really just need your brain, some paper, and a writing implement here (okay, and Matlab or similar).

2. Second, I encourage you to email me questions, ideas, comments, etc., about the class but request that you please do so in a respectful fashion.

3. Finally, as in all UVM classes, Academic honesty will be expected and departures will be dealt with appropriately. See http://www.uvm.edu/cses/ for guidelines.
Late policy: Unless in the case of an emergency (a real one) or if an absence has been predeclared and a make-up version sorted out, assignments that are not turned in on time or tests that are not attended will be given 0%.

Computing: Students are encouraged to use Matlab or something similar to check their work.

Note: for assignment problems, written details of calculations will be required.
Why are we doing this?

Big deal: **Linear Algebra** is a body of mathematics that deals with **discrete problems**.

Many things are discrete:

- Information (0’s & 1’s, letters, words)
- People (sociology)
- Networks (the Web, people again, food webs, ...)
- Sounds (musical notes)

Even more:

If real data is continuous, we almost always discretize it (0’s and 1’s)
Why are we doing this?

Linear Algebra is used in many fields to solve problems:

▶ Engineering
▶ Computer Science (Google’s Pagerank)
▶ Physics
▶ Economics
▶ Biology
▶ Ecology
▶ ...

Linear Algebra is as important as Calculus...

Calculus ≡ the blue pill...
You are now choosing the red pill:
Matrices as gadgets:

A matrix $A$ transforms a vector $\vec{x}$ into a new vector $\vec{x}'$ through matrix multiplication (whatever that is):

$$\vec{x}' = A \vec{x}$$

We can use matrices to:

- Grow vectors
- Shrink vectors
- Rotate vectors
- Flip vectors
- Do all these things in different directions
- Reveal the true ur-dystopian reality.
Three key problems of Linear Algebra

1. Given a matrix $A$ and a vector $\vec{b}$, find $\vec{x}$ such that

\[ A\vec{x} = \vec{b}. \]

2. Eigenvalue problem: Given $A$, find $\lambda$ and $\vec{v}$ such that

\[ A\vec{v} = \lambda\vec{v}. \]

3. Coupled linear differential equations:

\[ \frac{d}{dt}y(t) = A y(t) \]

- Our focus will be largely on #1, partly on #2.
Major course objective:

To deeply understand the equation $A\vec{x} = \vec{b}$, the Fundamental Theorem of Linear Algebra, and the following picture:

What is going on here? We have 25 lectures to find out...
Is this your left nullspace?:
Our new BFF: $A\vec{x} = \vec{b}$

Broadly speaking, $A\vec{x} = \vec{b}$ translates as follows:

- $\vec{b}$ represents reality (e.g., music, structure)
- $A$ contains building blocks (e.g., notes, shapes)
- $\vec{x}$ specifies how we combine our building blocks to make $\vec{b}$ (as best we can).

How can we disentangle an orchestra’s sound?

What about pictures, waves, signals, ...?
Our friend $A\vec{x} = \vec{b}$

What does knowing $\vec{x}$ give us?

If we can represent reality as a superposition (or combination or sum) of simple elements, we can do many things:

- Compress information
- See how we can alter information (filtering)
- Find a system’s simplest representation
- Find a system’s most important elements
- See how to adjust a system in a principled way
Three ways to understand $A\mathbf{x} = \mathbf{b}$:

- **Way 1:** The **Row** Picture
- **Way 2:** The **Column** Picture
- **Way 3:** The **Matrix** Picture

**Example:**

\[-x_1 + x_2 = 1
\]
\[2x_1 + x_2 = 4\]

- Call this a **2 by 2 system of equations**.
- **2** equations with **2 unknowns**.
- **Standard method of simultaneous equations:** solve above by adding and subtracting multiples of equations to each other = **Row Picture**.
Three ways to understand $A\vec{x} = \vec{b}$:

**Row Picture—what we are doing:**

- (a) Finding intersection of two lines
- (b) Finding the values of $x_1$ and $x_2$ for which both equations are satisfied (true/happy)
- A splendid and deep connection:
  - (a) Geometry $\iff$ (b) Algebra

**Three possible kinds of solution:**

1. Lines intersect at one point — **One, unique solution**
2. Lines are parallel and disjoint — **No solutions**
3. Lines are the same — **Infinitely many solutions**
Three ways to understand $A\vec{x} = \vec{b}$:

The column picture:

See

$$
\begin{align*}
-x_1 + x_2 &= 1 \\
2x_1 + x_2 &= 4
\end{align*}
$$

as

$$
\begin{align*}
x_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} &= \begin{bmatrix} 1 \\ 4 \end{bmatrix}.
\end{align*}
$$

General problem

$$
x_1 \vec{a}_1 + x_2 \vec{a}_2 = \vec{b}
$$

- Column vectors are our ‘building blocks’
- **Key idea:** try to ‘reach’ $\vec{b}$ by combining (summing) multiples of column vectors $\vec{a}_1$ and $\vec{a}_2$. 
Three ways to understand $A\vec{x} = \vec{b}$:

We love the column picture:

- Intuitive.
- Generalizes easily to many dimensions.

Three possible kinds of solution:

1. $\vec{a}_1 \parallel \vec{a}_2$: 1 solution
2. $\vec{a}_1 \parallel \vec{a}_2 \parallel \vec{b}$: No solutions
3. $\vec{a}_1 \parallel \vec{a}_2 \parallel \vec{b}$: infinitely many solutions

(assuming neither $\vec{a}_1$ or $\vec{a}_1$ are $\vec{0}$)
Three ways to understand $A\vec{x} = \vec{b}$:

**Difficulties:**
- Do we give up if $A\vec{x} = \vec{b}$ has no solution?
- **No!** We can still find the $\vec{x}$ that gets us as close to $\vec{b}$ as possible.
- Method of approximation—very important!
- We may not have the right building blocks but we can do our best.
Three ways to understand $A\vec{x} = \vec{b}$:

**The Matrix Picture:**

Now see

$$x_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$ 

as

$$A\vec{x} = \vec{b}: \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$ 

**A is now an operator:**

- $A$ transforms $\vec{x}$ into $\vec{b}$.
- Roughly speaking, $A$ does two things to $\vec{x}$:
  1. Rotation/Flipping
  2. Dilation (stretching/contraction)
Key idea in linear algebra:

- Decomposition or factorization of matrices.
- Matrices can often be written as products or sums of simpler matrices
  
  \[ A = LU, \quad A = QR, \quad A = U\Sigma V^T, \quad A = \sum_i \lambda_i \vec{v}_i \vec{v}_i^T, \ldots \]
The truth about mathematics
References

The fundamental theorem of linear algebra.

Too much calculus, 2002.
SIAM Linear Algebra Activity Group Newsletter. pdf