The law of first digits

Benford's Law:
- First observed by Simon Newcomb\(^2\) in 1881
  “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered by Frank Benford in 1938.
- Newcomb almost always noted but Benford gets the stamp
- $P(\text{first digit} = d) \propto \log_b (d + 1/d)$
  for numbers in base $b$

Benford's Law—The law of first digits

Observed for
- Fundamental constants (electron mass, charge, etc.)
- Utilities bills
- Numbers on tax returns
- Death rates
- Street addresses
- Numbers in newspapers
Benford's Law

Real data predicted frequencies


Essential story

- \( P(\text{first digit} = d) \propto \log_b (d + 1/d) \)
- \( P(\text{first digit} = d) \propto \log_b \left( \frac{d + 1}{d} \right) \)
- \( P(\text{first digit} = d) \propto \log_b (d + 1) - \log_b (d) \)
- So numbers are distributed uniformly in log-space:
  \[ P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx \]
- Independent of actual base and units of measurement.
- Power law distributions at work again... (\( \gamma = 1 \))

A different Benford

Not to be confused with Benford's Law of controversy:

- “Passion is inversely proportional to the amount of real information available.”

Gregory Benford, Sci-Fi writer & Astrophysicist

References I

- T. P. Hill.
  The first-digit phenomenon.
- S. Newcomb.
  Note on the frequency of use of the different digits in natural numbers.
  pdf ([link])