The amusing and excellent law of Benford

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Outline

Benford’s law

References
The law of first digits

Benford’s Law:

- First observed by Simon Newcomb\(^2\) in 1881 “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered by Frank Benford in 1938.
- Newcomb almost always noted but Benford gets the stamp

\[
P(\text{first digit} = d) \propto \log_b (d + 1/d)
\]

for numbers in base \(b\)
Benford’s Law—The law of first digits

Observed for

- Fundamental constants (electron mass, charge, etc.)
- Utilities bills
- Numbers on tax returns
- Death rates
- Street addresses
- Numbers in newspapers
Benford’s Law

Real data

Essential story

- \( P(\text{first digit} = d) \propto \log_b (d + 1 / d) \)

- \( P(\text{first digit} = d) \propto \log_b \left( \frac{d + 1}{d} \right) \)

- \( P(\text{first digit} = d) \propto \log_b (d + 1) - \log_b (d) \)

- So numbers are distributed uniformly in log-space:
  \[ P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx \]

- Independent of actual base and units of measurement.

- Power law distributions at work again... \( (\gamma = 1) \)
A different Benford

Not to be confused with Benford’s Law of controversy:

➤ “Passion is inversely proportional to the amount of real information available.”

Gregory Benford, Sci-Fi writer & Astrophysicist
References I

T. P. Hill.
The first-digit phenomenon.  

S. Newcomb.
Note on the frequency of use of the different digits in natural numbers.  

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