Definition:
- (1) The spreading of a quality or quantity between individuals in a population.
- (2) A disease itself: the plague, a blight, the dreaded lurgi, ...

Two main classes of contagion:
1. Infectious diseases: tuberculosis, HIV, ebola, SARS, influenza, ...
2. Social contagion: fashion, word usage, rumors, riots, religion, ...

Some large questions concerning network contagion:
1. For a given spreading mechanism on a given network, what’s the probability that there will be global spreading?
2. If spreading does take off, how far will it go?
3. How do the details of the network affect the outcome?
4. How do the details of the spreading mechanism affect the outcome?
5. What if the seed is one or many nodes?
Mathematical Epidemiology
The standard SIR model:
- Three states:
  - $S = \text{Susceptible}$
  - $I = \text{Infected}$
  - $R = \text{Recovered}$
- Presumes random interactions

Discrete time example:

Transition Probabilities:
- $\beta$ for being infected given contact with infected
- $r$ for recovery
- $\rho$ for loss of immunity

Disease spreading models

For 'novel' diseases:
1. Can we predict the size of an epidemic?
2. How important/useful is the reproduction number $R_0$?
3. What is the population size $N$?

Independent Interaction models

Reproduction Number $R_0$:
- $R_0 =$ expected number of infected individuals resulting from a single initial infective.
- Epidemic threshold: If $R_0 > 1$, ‘epidemic’ occurs.
- Example:

$R_0$ and variation in epidemic sizes

$R_0$ approximately the same for all of the following:
- 1918-19 “Spanish Flu” $\sim 500,000$ deaths in US
- 1957-58 “Asian Flu” $\sim 70,000$ deaths in US
- 1968-69 “Hong Kong Flu” $\sim 34,000$ deaths in US
- 2003 “SARS Epidemic” $\sim 800$ deaths world-wide
Elsewhere, event size distributions are important:

- earthquakes (Gutenberg-Richter law)
- city sizes, forest fires, war fatalities
- wealth distributions
- ‘popularity’ (books, music, websites, ideas)
- What about Epidemics?

Power laws distributions are common but not obligatory...

Insert plots:
Complementary cumulative frequency distributions:

\[ N(\psi) \propto \psi^{-\gamma+1} \]
\(\psi\) = fractional epidemic size

Measured values of \(\gamma\):

- measles: 1.40 (low \(\psi\)) and 1.13 (high \(\psi\))
- Expect 2 \(\leq \gamma < 3\) (finite mean, infinite variance)
- Distribution is rather flat...

Resurgence—example of SARS

Epidemic discovers new ‘pools’ of susceptibles: Resurgence.

Importance of rare, stochastic events.
A challenge

So... can a simple model produce
1. broad epidemic distributions
   and
2. resurgence?

A toy agent-based model

Geography: allow people to move between contexts:

- $P = \text{probability of travel}$
- Movement distance: $\Pr(d) \propto \exp(-d/\xi)$
- $\xi = \text{typical travel distance}$

Size distributions

Simple models typically produce bimodal or unimodal size distributions.

- This includes network models: random, small-world, scale-free, ...
- Some exceptions:
  1. Forest fire models
  2. Sophisticated metapopulation models

Example model output: size distributions

- Flat distributions are possible for certain $\xi$ and $P$.
- Different $R_0$’s may produce similar distributions
- Same epidemic sizes may arise from different $R_0$’s
Simple disease spreading models

Attempts to use beyond disease:
- Adoption of ideas/beliefs (Goffman & Newell, 1964)
- Spread of rumors (Daley & Kendall, 1965)
- Diffusion of innovations (Bass, 1969)
- Spread of fanatical behavior (Castillo-Chávez & Song, 2003)

Social Contagion

Examples abound:
- being polite/rude
- strikes
- innovation
- residential segregation
- ipods
- obesity
- Harry Potter
- voting
- gossip
- Rubik’s cube
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible
- Classes of behavior versus specific behavior: dieting
Social Contagion

Two focuses for us:
- Widespread media influence
- Word-of-mouth influence

The two step model of influence:

The general model of influence:
Social Contagion

Why do things spread?

- Because of system level properties?
- Or properties of special individuals?
- Is the match that lights the forest fire the key? (Katz and Lazarsfeld; Gladwell)
- Yes. But only because we are narrative-making machines...
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

The Mona Lisa:

- “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- Not the world’s greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

The completely unpredicted fall of Eastern Europe:


Social Contagion

Some important models:

- Tipping models—Schelling (1971)
  - Simulation on checker boards
  - Idea of thresholds
- Threshold models—Granovetter (1978)
- Herding models—Bikhchandani, Hirschleifer, Welch (1992)
  - Social learning theory, Informational cascades,...
Social contagion models

Thresholds:
- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- ‘Others’ may be everyone in a population, an individual’s close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds vary.

Some possible origins of thresholds:
- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
  - Telephones, Facebook, operating systems, ...

Imitation

“When people are free to do as they please, they usually imitate each other.”
—Eric Hoffer
“The Passionate State of Mind”[11]

Granovetter's threshold model:

Action based on perceived behavior of others:
- Two states: S and I.
- $\phi = \text{fraction of contacts ‘on’ (e.g., rioting)}$
- $\phi_{t+1} = \int_0^{\phi_t} f(\gamma) d\gamma = F(\gamma)|_{\phi_t}^{\phi_t} = F(\phi_t)$
- This is a Critical Mass model
Example of single stable state model

Threshold model on a network

- All nodes have threshold $\phi = 0.2$.
- “A simple model of global cascades on random networks”

Implications for collective action theory:

1. Collective uniformity $\neq$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes

Snowballing

The Cascade Condition:

- If one individual is initially activated, what is the probability that an activation will spread over a network?
- What features of a network determine whether a cascade will occur or not?
The most gullible

Vulnerables:
- ▶ Individuals who can be activated by just one ‘infected’ contact
- ▶ For global cascades on random networks, must have a *global cluster of vulnerables*
- ▶ Cluster of vulnerables = critical mass
- ▶ Network story: 1 node → critical mass → everyone.

Cascade window for random networks

- ▶ ‘Cascade window’ widens as threshold $\phi$ decreases.
- ▶ Lower thresholds enable spreading.
Analytic work

- Threshold model completely solved (by 2008):
  
  
  $$\sum_{k=1}^{\infty} k(k-1)\beta_k P_k / z \geq 1.$$

  
  where $\beta_k$ = probability a degree $k$ node is vulnerable.

- Final size of spread figured out by Gleeson and Calahane [9, 8].
- Solution involves finding fixed points of an iterative map of the interval.
- Spreading takes off: expansion
- Spreading reaches a particular node: contraction

Early adopters—degree distributions

$P_{k,t}$ versus $k$

The power of groups...

“A few harmless flakes working together can unleash an avalanche of destruction.”
Group structure—Ramified random networks

\[ p = \text{intergroup connection probability} \]
\[ q = \text{intragroup connection probability}. \]

Cascade windows for group-based networks

Generalized affiliation model

Assortativity in group-based networks

- The most connected nodes aren’t always the most ‘influential.’
- Degree assortativity is the reason.
Social Contagion

Summary:

- ‘Influential vulnerables’ are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Extreme/unexpected cascades may occur in highly connected networks.
- Many potential ‘influentials’ exist.
- Average individuals may be more influential system-wise than locally influential individuals.
- ‘Influentials’ are posterior constructs.

Implications:

- Focus on the influential vulnerables.
- Create entities that many individuals ‘out in the wild’ will adopt and display rather than broadcast from a few ‘influentials’.
- Displaying can be passive = free (yo-yo’s, fashion), or active = harder to achieve (political messages).
- Accept that movement of entities will be out of originator’s control.
- Possibly only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders has spread well...)

Messing with social connections:

- Ads based on message content (e.g., Google and email)
- Buzz media
- Facebook’s advertising (Beacon)

Arguably not always a good idea...

“Never Underestimate the Power of Stupid People in Large Groups.”

despair.com
Where do superstars come from?

Rosen (1981): “The Economics of Superstars”

Examples:
- Full-time Comedians (≈ 200)
- Soloists in Classical Music
- Economic Textbooks (the usual myopic example)
- Highly skewed distributions again...

Superstars

Adler (1985): “Stardom and Talent”
- Assumes extreme case of equal ‘inherent quality’
- Argues desire for coordination in knowledge and culture leads to differential success
- Success is then purely a social construction

Rosen’s theory:
- Individual quality $q$ maps to reward $R(q)$
- $R(q)$ is ‘convex’ ($d^2R/dq^2 > 0$)
- Two reasons:
  1. Imperfect substitution: A very good surgeon is worth many mediocre ones
  2. Technology: Media spreads & technology reduces cost of reproduction of books, songs, etc.
- No social element—success follows ‘inherent quality’

Dominance hierarchies

Chase et al. (2002): “Individual differences versus social dynamics in the formation of animal dominance hierarchies”

The aggressive female Metriaclima zebra (⋮):

Pecking orders for fish...
Fish forget—changing of dominance hierarchies:

- 22 observations: about 3/4 of the time, hierarchy changed

Table 1. Percentage of groups with different numbers of fish changing ranks between changing ranks between first and second hierarchies (

<table>
<thead>
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<th>No. of fish changing ranks</th>
<th>Percentage of groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2%</td>
</tr>
<tr>
<td>2</td>
<td>27.3%</td>
</tr>
<tr>
<td>3</td>
<td>3.2%</td>
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<td>16.7%</td>
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<tr>
<td>5</td>
<td>6.5%</td>
</tr>
<tr>
<td>&gt;5</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

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Variability in final rank.

Variability in final number of downloads.

Inequality as measured by Gini coefficient:

\[ G = \frac{1}{2N_s(N_s - 1)} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} |m_i - m_j| \]

Unpredictability

\[ U = \frac{1}{N_s(N_s - 1)^2} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \sum_{k=j+1}^{N_s} |m_{i,j} - m_{i,k}| \]
Music Lab Experiment

Sensible result:
- Stronger social signal leads to greater following and greater inequality.

Peculiar result:
- Stronger social signal leads to greater unpredictability.

Very peculiar observation:
- The most unequal distributions would suggest the greatest variation in underlying ‘quality.’
- But success may be due to social construction through following...

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