Some problems for sociologists

How are social networks structured?
- How do we define connections?
- How do we measure connections?
- (remote sensing, self-reporting)

What about the dynamics of social networks?
- How do social networks evolve?
- How do social movements begin?
- How does collective problem solving work?
- How is information transmitted through social networks?

Social Search

A small slice of the pie:
- Q. Can people pass messages between distant individuals using only their existing social connections?
- A. Apparently yes...

Handles:
- The Small World Phenomenon
- or “Six Degrees of Separation.”
The Small-World Phenomenon

History
An online experiment
Previous theoretical work
An improved model
References

The problem

Stanley Milgram et al., late 1960’s:

- Target person worked in Boston as a stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- average chain length \( \approx 6.5 \).

From Travers and Milgram (1969) in Sociometry: [4]

“An Experimental Study of the Small World Problem.”

The problem

Lengths of successful chains:

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Social Search

Milgram’s small world experiment with e-mail [2]

Two features characterize a social ‘Small World’:

1. Short paths exist
   and
2. People are good at finding them.
Social search—the Columbia experiment

- 60,000+ participants in 166 countries
- 18 targets in 13 countries including:
  - a professor at an Ivy League university,
  - an archival inspector in Estonia,
  - a technology consultant in India,
  - a policeman in Australia,
  and
  - a veterinarian in the Norwegian army.
- 24,000+ chains

Social search—the Columbia experiment

- Milgram’s participation rate was roughly 75%
- Email version: Approximately 37% participation rate.
- Probability of a chain of length 10 getting through:
  \[ 0.37^{10} \approx 5 \times 10^{-5} \]
- \[ \Rightarrow 384 \text{ completed chains (1.6\% of all chains)}. \]

Social search—the Columbia experiment

Successful chains disproportionately used
- weak ties (Granovetter)
- professional ties (34\% vs. 13\%)
- ties originating at work/college
- target’s work (65\% vs. 40\%)

\[ \text{\ldots and disproportionately avoided} \]
- hubs (8\% vs. 1\%) (+ no evidence of funnels)
- family/friendship ties (60\% vs. 83\%)

Geography → Work

Social search—the Columbia experiment

Motivation/Incentives/Perception matter.
- If target seems reachable
  \[ \Rightarrow \text{participation more likely}. \]
- Small changes in attrition rates
  \[ \Rightarrow \text{large changes in completion rates}. \]
- e.g., \[ \downarrow 15\% \text{ in attrition rate} \]
  \[ \Rightarrow \uparrow 800\% \text{ in completion rate} \]
Social search—the Columbia experiment

Senders of successful messages showed little absolute dependency on
- age, gender
- country of residence
- income
- religion
- relationship to recipient

Range of completion rates for subpopulations: 30% to 40%

Social search—the Columbia experiment

Nevertheless, some weak discrepancies do exist...

An above average connector:
Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:
Italian, Islamic or Christian female earning less than $2K, with elementary school education and retired, who uses strong ties to family members.

Mildly bad for continuing chain:
choosing recipients because “they have lots of friends” or because they will “likely continue the chain.”

Why:
- Specificity important
- Successful links used relevant information.
  (e.g. connecting to someone who shares same profession as target.)

Basic results:
- $\langle L \rangle = 4.05$ for all completed chains
- $L_* = \text{Estimated ‘true’ median chain length (zero attrition)}$
- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$
- Milgram: $L_* \simeq 9$
Previous work—short paths

Connected random networks have short average path lengths:
\[ \langle d_{AB} \rangle \sim \log(N) \]

\( N \) = population size,
\( d_{AB} \) = distance between nodes \( A \) and \( B \).

But: social networks aren’t random...

Non-randomness gives clustering

\[ d_{AB} = 10 \rightarrow \text{too many long paths.} \]

Randomness + regularity

Now have \( d_{AB} = 3 \)

\( \langle d \rangle \) decreases overall
Small-world networks


Small-world networks were found everywhere:
- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph,
- food webs,
- social networks of comic book characters,…

Very weak requirements:
- local regularity + random short cuts

The structural small-world property

Previous work—finding short paths

But are these short cuts findable?

No.

Nodes cannot find each other quickly with any local search method.
Previous work—finding short paths

What can a local search method reasonably use?
How to find things without a map?
Need some measure of distance between friends and the target.

Some possible knowledge:
- Target's identity
- Friends' popularity
- Friends' identities
- Where message has been

Previous work—finding short paths

“Navigation in a small world.”

Allowed to vary:
1. local search algorithm and
2. network structure.

Previous work—finding short paths

Kleinberg's Network:
1. Start with regular d-dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance $q$.
3. Add $m$ short cuts per node.
4. Connect $i$ to $j$ with probability

$$p_{ij} \propto d_{ij}^{-\alpha}.$$ 

- $\alpha = 0$: random connections.
- $\alpha$ large: reinforce local connections.
- $\alpha = d$: same number of connections at all scales.

Previous work—finding short paths

Theoretical optimal search:
- “Greedy” algorithm.
- Same number of connections at all scales: $\alpha = d$.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.
Previous work—finding short paths

- If networks have **hubs** can also search well: Adamic et al. (2001) \(^1\)
  \[ P(k_i) \propto k_i^{-\gamma} \]
  where \( k = \text{degree of node } i \) (number of friends).
- Basic idea: get to hubs first (airline networks).
- But: hubs in social networks are limited.

**The problem**

If there are no hubs and no underlying lattice, how can search be efficient?

Which friend of \( a \) is closest to the target \( b \)?

What does ‘closest’ mean?

What is ‘social distance’?

**The model**

One approach: incorporate **identity**. (See “Identity and Search in Social Networks.” Science, 2002, Watts, Dodds, and Newman \(^2\))

**Identity** is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

**Groups** are formed by people with at least one similar attribute.

**Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.**

### Social distance—Bipartite affiliation networks

```
1 2 3 4
a b c d e
```

\( \text{contexts} \)

\( \text{individuals} \)

\( \text{unipartite network} \)
The model

- Individuals are more likely to know each other the closer they are within a hierarchy.
- Construct $z$ connections for each node using
  \[ p_{ij} = c \exp\{-\alpha x_{ij}\}. \]
  - $\alpha = 0$: random connections.
  - $\alpha$ large: local connections.

Distance between two individuals $x_{ij}$ is the height of lowest common ancestor.

\[ x_{ij} = 3, \ x_{ik} = 1, \ x_{iv} = 4. \]
The model

The model

The model-results—searchable networks

\begin{align*}
\vec{v}_i &= [1 \ 1 \ 1]^T, \quad \vec{v}_j = [8 \ 4 \ 1]^T \\
& y_{ij} = \min_h x_{ij}^h.
\end{align*}

Triangle inequality doesn’t hold:

\begin{align*}
y_{ik} &= 4 > y_{ij} + y_{jk} = 1 + 1 = 2.
\end{align*}

▶ Individuals know the identity vectors of
  1. themselves,
  2. their friends, and
  3. the target.
▶ Individuals can estimate the social distance between their friends and the target.
▶ Use a greedy algorithm + allow searches to fail randomly.

$q = \text{probability an arbitrary message chain reaches a target.}$

▶ A few dimensions help.
▶ Searchability decreases as population increases.
▶ Precise form of hierarchy largely doesn’t matter.
The model-results

Milgram’s Nebraska-Boston data:

Model parameters:
- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;
- $\langle L_{model} \rangle \simeq 6.7$
- $L_{data} \simeq 6.5$

Social search—Data

Adamic and Adar (2003)
- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.

Social Search—Real world uses

- Tags create identities for objects
- Website tagging: http://www.del.icio.us (e.g., Wikipedia)
- Photo tagging: http://www.flickr.com
- Dynamic creation of metadata plus links between information objects.
- Folksonomy: collaborative creation of metadata

Social Search—Real world uses

Recommender systems:
- Amazon uses people’s actions to build effective connections between books.
- Conflict between ‘expert judgments’ and tagging of the hoi polloi.
Conclusions

- Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- Importance of identity (interaction contexts).
- Improved social network models.
- Construction of peer-to-peer networks.
- Construction of searchable information databases.

References

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