Chapter 2: Lecture 7
Linear Algebra, Course 124C, Spring, 2009

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Basics:

Sections covered on first midterm:

- Chapter 1 and Chapter 2 (Sections 2.1–2.7)
- Chapter 2 is our focus
- Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- Want ‘understanding’ and ‘doing’ abilities.
Stuff to know:

Row, Column, & Matrix Pictures of Linear Systems 
\((A\vec{x} = \vec{b})\)

- What dimensions of \(A\) mean:
  - \(m = \) number of equations
  - \(n = \) number of unknowns \((x_1, x_2, \ldots)\)

- How to draw the row and column pictures.
- Be able to identify row picture
  (e.g., as representing 2 planes in 3-d).
- How to convert between the three pictures.
Solving $A\vec{x} = \vec{b}$ by elimination

Solve four equivalent ways:

1. Simultaneous equations (snore)
2. Row operations on augmented matrix
   - Systematically transform $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$
   - Solve by back substitution
3. Row operations with $E_{ij}$ and $P_{ij}$ matrices
4. Factor $A$ as $A = LU$
   - Solve two triangular systems by forward and back substitution
     - First $L\vec{c} = \vec{b}$ then $U\vec{x} = \vec{c}$.

Understand number of solutions business:

- 0, 1, or $\infty$: why, when, ...
Stuff to know:

More on $A = LU$:

- Be able to find the pivots of $A$ (they live in $U$)
- Understand how elimination matrices ($E_{ij}$’s) are constructed from multipliers ($l_{ij}$’s)
- Understand how $L$ is made up of inverses of elimination matrices
  - e.g.: $L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} A$.
- Understand how $L$ is made up of the $l_{ij}$ multipliers.
- Understand how inverses of elimination matrices are simply related to elimination matrices.
Stuff to know:

Matrix algebra

- Understand basic matrix algebra
- Understand matrix multiplication
- Understand multiplication order matters
- Understand $AB = BA$ is rarely true

Inverses

- Understand identity matrix $I$
- Understand $AA^{-1} = A^{-1}A = I$
- Find $A^{-1}$ with Gauss-Jordan elimination
- Perform row reduction on augmented matrix $[A|I]$.
- Understand that finding $A^{-1}$ solves $A\vec{x} = \vec{b}$ but is often prohibitively expensive to do.
- $(AB)^{-1} = B^{-1}A^{-1}$
Stuff to know:

Transposes

- Definition: flip entries across main diagonal
- $A = A^T$: $A$ is symmetric
- Important property: $(AB)^T = B^T A^T$

Extra pieces:

- If $A\vec{x} = \vec{0}$ has a non-zero solution, $A$ has no inverse
- If $A\vec{x} = \vec{0}$ has a non-zero solution, then $A\vec{x} = \vec{b}$ always has infinitely many solutions.
- $(A^{-1})^T = (A^T)^{-1}$