Social Contagion

Principles of Complex Systems
Course 300, Fall, 2008

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Social Contagion Models
  Background
  Granovetter’s model
  Network version
  Groups
  Chaos

References
Outline

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  Background
    Granovetter’s model
    Network version
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  Chaos

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Social Contagion

Examples abound

- fashion
- striking
- smoking \(^{[6]}\)
- residential segregation \(^{[15]}\)
- ipods
- obesity \(^{[5]}\)

- Harry Potter
- voting
- gossip
- Rubik’s cube
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior
Social Contagion

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- striking
- smoking (田) [6]
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- striking
- smoking \[^6\]
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- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior: dieting
Framingham heart study:

Evolving network stories:

- The spread of **quitting smoking** (込) [6]
- The spread of **spreading** (込) [5]
Social Contagion

Two focuses for us

▶ Widespread media influence
▶ Word-of-mouth influence
Social Contagion

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Two focuses for us

- Widespread media influence
- Word-of-mouth influence
We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [12]
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- The infectious idea of opinion leaders (Katz and Lazarsfeld) [12]
The hypodermic model of influence
The two step model of influence [12]
The general model of influence
Social Contagion

Why do things spread?

- Because of system level properties?
- Or properties of special individuals?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...
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The Mona Lisa

- “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- Not the world’s greatest painting from the start...
- Escalation through theft, vandalism,
The Mona Lisa

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Not the world’s greatest painting from the start...

Escalation through theft, vandalism, parody, ...
The completely unpredicted fall of Eastern Europe

The dismal predictive powers of editors...
Social Contagion

Messing with social connections

- Ads based on message content
- Buzz media
- Facebook’s advertising: Beacon (☞)
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- Ads based on message content
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Social Contagion

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- Ads based on message content (e.g., Google and email)
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Getting others to do things for you


Six modes of influence

1. Reciprocation: The Old Give and Take... and Take
2. Commitment and Consistency: Hobgoblins of the Mind
3. Social Proof: Truths Are Us
4. Liking: The Friendly Thief
5. Authority: Directed Deference
6. Scarcity: The Rule of the Few
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Examples

- **Reciprocation**: Free samples, Hare Krishnas
- **Commitment and Consistency**: Hazing
- **Social Proof**: Catherine Genovese, Jonestown
- **Liking**: Separation into groups is enough to cause problems.
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Getting others to do things for you

- Cialdini’s modes are heuristics that help us get through life.
- Useful but can be leveraged...
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Other acts of influence

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)
Social Contagion

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Some important models

- **Tipping models—Schelling (1971)** \[15, 16, 17\]
  - Simulation on checker boards
  - Idea of thresholds
  - Fun with Netlogo and Schelling’s model \[20\]...

- **Threshold models—Granovetter (1978)** \[9\]

- **Herding models—Bikhchandani, Hirschleifer, Welch (1992)** \[1, 2\]

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  - Social learning theory, Informational cascades,...
Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted.
- ‘Others’ may be everyone in a population, an individual’s close friends, or any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary.
- Assumption: order of others’ adoption does not matter...
- Assumption: level of influence per person is uniform.
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- Response can be probabilistic or deterministic.
- Individual thresholds can vary.
- Assumption: order of others’ adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).
Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual’s utility increases with the adoption level among peers and the population in general
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- $\phi^* = \text{threshold of an individual.}$
- $f(\phi^*) = \text{distribution of thresholds in a population.}$
- $F(\phi^*) = \text{cumulative distribution} = \int_{\phi^*}^{\phi^*} f(\phi^*)d\phi^*$
- $\phi_t = \text{fraction of people ‘rioting’ at time step } t.$
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Threshold models

- Example threshold influence response functions: deterministic and stochastic
  - $\phi = \text{fraction of contacts ‘on’ } \text{(e.g., rioting)}$
  - Two states: S and I.
Threshold models

- Example threshold influence response functions: deterministic and stochastic
- $\phi = \text{fraction of contacts ‘on’ (e.g., rioting)}$
- Two states: S and I.
Example threshold influence response functions: deterministic and stochastic

\( \phi = \text{fraction of contacts ‘on’ (e.g., rioting)} \)

Two states: S and I.
Threshold models

- At time $t + 1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.

$$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*) d\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$

⇒ Iterative maps of the unit interval $[0, 1]$. 
Threshold models

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- Imply iterative maps of the unit interval $[0, 1]$. 
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Action based on perceived behavior of others.

- Two states: S and I.
- $\phi = \text{fraction of contacts ‘on’ (e.g., rioting)}$
- Discrete time update (strong assumption!)
- This is a Critical mass model
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▶ Another example of critical mass model...
Threshold models

Example of single stable state model
Threshold models

Implications for collective action theory:

1. Collective uniformity $\nrightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes
Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\approx$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes
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Chaotic behavior possible [11, 10]

- Period doubling arises as map amplitude $r$ is increased.
- Synchronous update assumption is crucial.
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- Mean field model → network model
- Individuals now have a limited view of the world
Many years after Granovetter and Soong’s work:

“A simple model of global cascades on random networks”


- Mean field model → network model
- Individuals now have a limited view of the world
Threshold model on a network

Many years after Granovetter and Soong’s work:

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- Mean field model $\rightarrow$ network model
- Individuals now have a limited view of the world
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- Interactions between individuals now represented by a network
- Network is sparse
- Individual $i$ has $k_i$ contacts
- Influence on each link is reciprocal and of unit weight
- Each individual $i$ has a fixed threshold $\phi_i$
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual $i$ becomes active when fraction of active contacts $a_i \geq \phi_i k_i$
- Individuals remain active when switched (no recovery = SI model)
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- All nodes have threshold $\phi = 0.2$. 

![Diagram](image-url)
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The Cascade Condition:

If one individual is initially activated, what is the probability that an activation will spread over a network?
Snowballing

The Cascade Condition:

If one individual is initially activated, what is the probability that an activation will spread over a network?

What features of a network determine whether a cascade will occur or not?
First study random networks:

- Start with $N$ nodes with a degree distribution $p_k$
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition
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Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neighbors becomes active.
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The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active **vulnerables**
- The vulnerability condition for node $i$:
  \[
  \frac{1}{k_i} \geq \phi_i
  \]
  - Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
  - For global cascades on random networks, must have a **global cluster of vulnerables**\(^{[19]}\)
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.
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Back to following a link:

- Link from leads to a node with probability \( \propto kP_k \).
- Follows from links being random + having \( k \) chances to connect to a node with degree \( k \).
- Normalization:

\[
\sum_{k=0}^{\infty} kP_k = \langle k \rangle = z
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- So

\[
P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}
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Next: Vulnerability of linked node

- Linked node is **vulnerable** with probability
  \[
  \beta_k = \int_{\phi_*=0}^{1/k} f(\phi'_*)d\phi'_*
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- If linked node is **vulnerable**, it produces \(k - 1\) new outgoing active links
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Putting things together:

- Expected number of active edges produced by an active edge =

\[
\sum_{k=1}^{\infty} (k - 1) \beta_k \frac{k P_k}{z} + \\
= \sum_{k=1}^{\infty} (k - 1) k \beta_k P_k / z
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Cascade condition

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Cascade condition

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} k(k - 1)\beta_k P_k / z \geq 1.$$ 

- $\beta_k = \text{probability a degree } k \text{ node is vulnerable.}$
- $P_k = \text{probability a node has degree } k.$
Cascade condition

Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

\[
\beta \sum_{k=1}^{\infty} k(k - 1) P_k/z \geq 1.
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- (2) Giant component exists: $\beta = 1$

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Cascades on random networks

- Cascades occur only if size of max vulnerable cluster > 0.
- System may be ‘robust-yet-fragile’.
- ‘Ignorance’ facilitates spreading.
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Example networks
Cascades occur only if size of max vulnerable cluster > 0.

System may be ‘robust-yet-fragile’.

‘Ignorance’ facilitates spreading.
Cascade window for random networks

- ‘Cascade window’ widens as threshold $\phi$ decreases.
- Lower thresholds enable spreading.
Cascade window for random networks
Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.

2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.

3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in “Cascade window.”
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All-to-all versus random networks

**A.** all-to-all networks

**B.** random networks

**C.**

**D.**
Early adopters—degree distributions

\[ P_{k,t} \text{ versus } k \]
Early adopters—degree distributions

$t = 0$

$t = 1$

$t = 2$

$t = 3$

$t = 4$

$t = 6$

$t = 8$

$t = 10$

$t = 12$

$t = 14$

$t = 16$

$t = 18$

$P_{k,t}$ versus $k$
Early adopters—degree distributions

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Early adopters—degree distributions

$P_{k,t}$ versus $k$
The multiplier effect:

- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.
The multiplier effect:

- Skewed influence distribution example.
Special subnetworks can act as triggers

ϕ = 1/3 for all nodes
Outline

Social Contagion Models
  Background
  Granovetter’s model
  Network version
Groups
Chaos

References
The power of groups...

“A few harmless flakes working together can unleash an avalanche of destruction.”

despair.com
Extensions

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network’s function
- Still, random networks don’t represent all networks
- Major element missing: group structure
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Group structure—Ramified random networks

\[ p = \text{intergroup connection probability} \]
\[ q = \text{intragroup connection probability}. \]
Bipartite networks

- Contexts: 1, 2, 3, 4
- Individuals: a, b, c, d, e

- Unipartite network:
  - a → b
  - b → c
  - c → d
  - d → e
Context distance

- Occupation
  - Education
    - High school teacher
    - Kindergarten teacher
  - Health care
    - Nurse
    - Doctor

References
Generalized affiliation model

(Blau & Schwartz, Simmel, Breiger)
Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto \exp^{-\alpha d}$
  where
  $\alpha = \text{homophily parameter}$
  and
  $d = \text{distance between nodes (height of lowest common ancestor)}$

- $\tau_1 = \text{intergroup probability of friend-of-friend connection}$

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Cascade windows for group-based networks

- Single seed
- Random set seed
- Coherent group seed

Random Group networks

Generalized Affiliation

Model networks

References
Multipler effect for group-based networks:

- Multiplier almost always below 1.
Assortativity in group-based networks

The most connected nodes aren’t always the most ‘influential.’

Degree assortativity is the reason.
Assortativity in group-based networks

The most connected nodes aren’t always the most ‘influential.’

Degree assortativity is the reason.
Social contagion

Summary

- ‘Influential vulnerability’ are key to spread.
- Early adopters are mostly vulnerable.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks.
- ‘Influentials’ are posterior constructs.
- Many potential influentials exist.
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Social contagion

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Implications

- Focus on the influential vulnerables.
  - Create entities that can be transmitted successfully through many individuals rather than broadcast from one ‘influential.’
  - Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
  - Want enough individuals who will adopt and display.
  - Displaying can be passive = free (yo-yo’s, fashion), or active = harder to achieve (political messages).
  - Entities can be novel or designed to combine with others, e.g. block another one.
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Social Contagion Models

Background
Granovetter’s model
Network version
Groups
Chaos
Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:
  - Node $i$ has an ‘activation threshold’ $\phi_{i,1}$
  - ... and a ‘de-activation threshold’ $\phi_{i,2}$
  - Nodes like to imitate but only up to a limit—they don’t want to be like everyone else.
Chaotic contagion:

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Two population examples:

- Randomly select \((\phi_{i,1}, \phi_{i,2})\) from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We’ll consider plot C’s example: the tent map.
Two population examples:

- Randomly select \((\phi_{i,1}, \phi_{i,2})\) from gray regions shown in plots B and C.

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- We’ll consider plot C’s example: the tent map.
Chaotic contagion

Definition of the tent map:

\[ F(x) = \begin{cases} 
  rx & \text{for } 0 \leq x \leq \frac{1}{2}, \\
  r(1 - x) & \text{for } \frac{1}{2} \leq x \leq 1.
\end{cases} \]

- The usual business: look at how \( F \) iteratively maps the unit interval \([0, 1]\).
The tent map

Effect of increasing $r$ from 1 to 2.
The tent map

Effect of increasing $r$ from 1 to 2.
The tent map

Effect of increasing $r$ from 1 to 2.

![Graphs showing the effect of increasing $r$ from 1 to 2 on the tent map.](image)
The tent map

Effect of increasing $r$ from 1 to 2.

Orbit diagram:
Chaotic behavior increases as map slope $r$ is increased.
Chaotic behavior

Take \( r = 2 \) case:

- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree \( z = \langle k \rangle \), a measure of information
Chaotic behavior

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Invariant densities—stochastic response functions

activation time series

activation density

$z = 5$
Invariant densities—stochastic response functions

Frame 76/88
Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$
Invariant densities—stochastic response functions

Trying out higher values of $\langle k \rangle \ldots$
Invariant densities—deterministic response functions

Trying out higher values of $\langle k \rangle$...
Connectivity leads to chaos:

Stochastic response functions:
Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

\[ x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n}) \]

- \( \mathcal{N}_i = \) neighborhood of node \( i \)

1. Node states are continuous
2. Increase \( \delta \) and neighborhood size \( |\mathcal{N}| \) \[ \Rightarrow \text{synchronization} \]

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases
Chaotic behavior in coupled systems

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Bifurcation diagram: Asynchronous updating

\[ P(s | r) / \max P(s | r) \]

\[ r \]

\[ \alpha \]

\[ P(s | r) \]
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