Benford’s law—The law of first digits

- First observed by Simon Newcomb\(^2\) in 1881 “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered by Frank Benford in 1938.
- Newcomb almost always noted but Benford gets the stamp

\[ P(\text{first digit} = d) \propto \log_b (d + 1/d) \]

for numbers is base \(b\)

Benford’s Law—The law of first digits

Observed for
- Fundamental constants (electron mass, charge, etc.)
- Utilities bills
- Numbers on tax returns
- Death rates
- Street addresses
- Numbers in newspapers
Benford's law

Real data

![Graph showing frequency of first digits]


A different Benford

Not to be confused with Benford's law of controversy:

- “Passion is inversely proportional to the amount of real information available.”

Gregory Benford, Sci-Fi writer & Astrophysicist

Essential story

- \[ P(\text{first digit} = d) \propto \log_b (d + 1/d) \]
- \[ P(\text{first digit} = d) \propto \log_b \left( \frac{d + 1}{d} \right) \]
- So numbers are distributed uniformly in log-space:
  \[ P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx \]
- Power law distributions at work again... (\( \gamma = 1 \))

References I


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