The amusing and excellent law of Benford

Principles of Complex Systems
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Outline

Benford’s law

References
Benford’s law—The law of first digits

- First observed by Simon Newcomb \(^\text{[2]}\) in 1881 “Note on the Frequency of Use of the Different Digits in Natural Numbers”
- Independently discovered by Frank Benford in 1938.
- Newcomb almost always noted but Benford gets the stamp

\[
P(\text{first digit } = d) \propto \log_b (d + 1/d)
\]

for numbers is base \(b\)
Benford’s Law—The law of first digits

Observed for

- Fundamental constants (electron mass, charge, etc.)
- Utilities bills
- Numbers on tax returns
- Death rates
- Street addresses
- Numbers in newspapers
Benford’s law

Real data

Essential story

\[ P(\text{first digit} = d) \propto \log_b (d + 1/d) \]

\[ P(\text{first digit} = d) \propto \log_b \left( \frac{d + 1}{d} \right) \]

\[ P(\text{first digit} = d) \propto \log_b (d + 1) - \log_b (d) \]

So numbers are distributed uniformly in log-space:

\[ P(\ln x) \, d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} \, dx \]

Power law distributions at work again... \((\gamma = 1)\)
A different Benford

Not to be confused with Benford’s law of controversy:

- “Passion is inversely proportional to the amount of real information available.”

Gregory Benford, Sci-Fi writer & Astrophysicist
References

- **T. P. Hill.**
  The first-digit phenomenon.

- **S. Newcomb.**
  Note on the frequency of use of the different digits in natural numbers.
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