Chapter 6: Lecture 25
Linear Algebra, Course 124B, Fall, 2008

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Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
  The basic idea
  Guess who?
  Bonus example 1
  Bonus example 2
All the way with $A\vec{x} = \vec{b}$:
All the way with $A\vec{x} = \vec{b}$:

- Applies to any $m \times n$ matrix $A$.
- Symmetry of $A$ and $A^T$. 
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Where $\vec{x}$ lives:
All the way with $A\vec{x} = \vec{b}$:

- Applies to any $m \times n$ matrix $A$.
- Symmetry of $A$ and $A^T$.

Where $\vec{x}$ lives:

- Row space $C(A^T) \subset R^n$.
- (Right) Nullspace $N(A) \subset R^n$. 

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2
All the way with $A\vec{x} = \vec{b}$:

- Applies to any $m \times n$ matrix $A$.
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Where $\vec{b}$ lives:
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- Applies to any $m \times n$ matrix $A$.
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Where $\vec{x}$ lives:

- Row space $C(A^T) \subset \mathbb{R}^n$.
- (Right) Nullspace $N(A) \subset \mathbb{R}^n$.

Where $\vec{b}$ lives:

- Column space $C(A) \subset \mathbb{R}^m$.
- Left Nullspace $N(A^T) \subset \mathbb{R}^m$. 
All the way with $A\vec{x} = \vec{b}$:

- Applies to any $m \times n$ matrix $A$.
- Symmetry of $A$ and $A^T$.

Where $\vec{x}$ lives:

- Row space $C(A^T) \subset \mathbb{R}^n$.
- (Right) Nullspace $N(A) \subset \mathbb{R}^n$.
- $\dim C(A^T) + \dim N(A) = r + (n - r) = n$

Where $\vec{b}$ lives:

- Column space $C(A) \subset \mathbb{R}^m$.
- Left Nullspace $N(A^T) \subset \mathbb{R}^m$. 
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- Row space $C(A^T) \subset \mathbb{R}^n$.
- (Right) Nullspace $N(A) \subset \mathbb{R}^n$.
- $\dim C(A^T) + \dim N(A) = r + (n - r) = n$
- Orthogonality: $C(A^T) \bigotimes N(A) = \mathbb{R}^n$

Where $\vec{b}$ lives:

- Column space $C(A) \subset \mathbb{R}^m$.
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- $\dim C(A^T) + \dim N(A) = r + (n - r) = n$
- Orthogonality: $C(A^T) \perp N(A) = R^n$

Where $\vec{b}$ lives:

- Column space $C(A) \subset R^m$.
- Left Nullspace $N(A^T) \subset R^m$.
- $\dim C(A) + \dim N(A^T) = r + (m - r) = m$
- Orthogonality: $C(A) \perp N(A^T) = R^m$
Best solution $\vec{x}_*$ when $\vec{b} = \vec{p} + \vec{e}$:
Fundamental Theorem of Linear Algebra

Now we see:

- Each of the four fundamental subspaces has a ‘best’ orthonormal basis
Fundamental Theorem of Linear Algebra

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- The $\hat{v}_i$ span $\mathbb{R}^n$
Fundamental Theorem of Linear Algebra

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- The $\hat{v}_i$ span $\mathbb{R}^n$
- We find the $\hat{v}_i$ as eigenvectors of $A^T A$. 
Fundamental Theorem of Linear Algebra

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- The \( \hat{v}_i \) span \( R^n \)
- We find the \( \hat{v}_i \) as eigenvectors of \( A^T A \).
- The \( \hat{u}_i \) span \( R^m \)
Fundamental Theorem of Linear Algebra

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- Each of the four fundamental subspaces has a ‘best’ orthonormal basis.
- The $\hat{v}_i$ span $R^n$.
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- We find the $\hat{u}_i$ as eigenvectors of $AA^T$. 
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Happy bases

- $\{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
Fundamental Theorem of Linear Algebra

Now we see:

- Each of the four fundamental subspaces has a ‘best’ orthonormal basis
- The $\hat{v}_i$ span $\mathbb{R}^n$
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Happy bases

- $\{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \ldots, \hat{v}_n\}$ span Null space
Fundamental Theorem of Linear Algebra

Now we see:

- Each of the four fundamental subspaces has a ‘best’ orthonormal basis
  - The $\hat{v}_i$ span $R^n$
  - We find the $\hat{v}_i$ as eigenvectors of $A^T A$.
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Happy bases

- $\{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
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- $\{\hat{u}_1, \ldots, \hat{u}_r\}$ span Column space
Fundamental Theorem of Linear Algebra

Now we see:

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- The \( \hat{v}_i \) span \( R^n \)
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Happy bases

- \( \{ \hat{v}_1, \ldots, \hat{v}_r \} \) span Row space
- \( \{ \hat{v}_{r+1}, \ldots, \hat{v}_n \} \) span Null space
- \( \{ \hat{u}_1, \ldots, \hat{u}_r \} \) span Column space
- \( \{ \hat{u}_{r+1}, \ldots, \hat{u}_m \} \) span Left Null space
Fundamental Theorem of Linear Algebra

How $A\vec{x}$ works:
Fundamental Theorem of Linear Algebra

How $A \vec{x}$ works:

1. $A = U \Sigma V^T$
Fundamental Theorem of Linear Algebra

How $A\vec{x}$ works:

- $A = U\Sigma V^T$
- $A$ sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$. 
Fundamental Theorem of Linear Algebra

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- $A$ is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
Fundamental Theorem of Linear Algebra

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- $A = U\Sigma V^T$
- $A$ sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- $A$ is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any $A$ is a diagonal matrix $\Sigma$. 
Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
  The basic idea
  Guess who?
  Bonus example 1
  Bonus example 2
Image approximation (80x60)

Idea: use SVD to approximate images

- Interpret elements of matrix $A$ as color values of an image.

\[
A = U \Sigma V^T = \sum_{i=1}^{\min(m, n)} \sigma_i \hat{u}_i \hat{v}_i^T
\]

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- Rank $r = \# \text{ of pixels on shortest side}$.
- For color: approximate 3 matrices (RGB).
Image approximation (80x60)

Idea: use SVD to approximate images

► Interpret elements of matrix $A$ as color values of an image.

► Truncate series SVD representation of $A$:

$$A = U\Sigma V^T = \sum_{i=1}^{r} \sigma_i \hat{u}_i \hat{v}_i^T$$
Image approximation (80x60)

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Image approximation (80x60)

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Image approximation (80x60)

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- Rank $r =$ # of pixels on shortest side.
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Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
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  Bonus example 1
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The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (80x60)

\[ A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra
Approximating matrices with SVD
The basic idea
Guess who?
Bonus example 1
Bonus example 2

\[ A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (80x60)

\[ A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T \]
Decay of sigma values: Einstein
Image approximation (480x615)

\[ A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x615)

\[ A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x615)

\[ A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{7} \sigma_i \hat{u}_i^{\top} \hat{v}_i \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation \((480x615)\)

\[
A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^T
\]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x615)

\[ A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation ($480 \times 615$)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x615)

\[ A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x615)

\[ A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T \]
Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
  The basic idea
  Guess who?
  Bonus example 1
  Bonus example 2
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1
Bonus example 2

\[ A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1
Bonus example 2

Image approximation (480x640)

\[ A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T$$
Image approximation (480x640)

\[ A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T \]

The fundamental theorem of linear algebra
Approximating matrices with SVD
The basic idea
Guess who?
Bonus example 1
Bonus example 2
Image approximation (480x640)

\[ A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T \]
Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
  The basic idea
  Guess who?
  Bonus example 1

Bonus example 2
Image approximation (480x640)

\[ A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^T$$
Image approximation (480x640)

\[ A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[
A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^T
\]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x640)

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A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^T
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Image approximation (480x640)

\[ A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T \]
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Image approximation (480x640)

\[ A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$
The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T

Bonus example 1

Bonus example 2

Image approximation (480x640)
Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T$$
Image approximation (480x640)

\[ A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T \]
Image approximation (480x640)

\[ A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T \]