Applications of Random Networks
Complex Networks, Course 295A, Spring, 2008

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More on building random networks

► **Problem:** How much of a real network’s structure is non-random?
► Key elephant in the room: the degree distribution $P_k$.
► First observe departure of $P_k$ from a Poisson distribution.
► **Next:** measure the departure of a real network with a degree frequency $N_k$ from a random network with the same degree frequency.
► Degree frequency $N_k$ = observed frequency of degrees for a real network.
► **What we now need to do:** Create an ensemble of random networks with degree frequency $N_k$ and then compare.

Building random networks: Stubs

**Phase 1:**

► **Idea:** start with a soup of unconnected nodes with stubs (half-edges):

```
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
      ●  ●
```

► Randomly select stubs (not nodes!) and connect them.
► Must have an even number of stubs.
► Initially allow self- and repeat connections.
Building random networks: First rewiring

Phase 2:
- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.
- Being careful: we can’t change the degree of any node, so we can’t simply move links around.
- Simplest solution: randomly rewire two edges at a time.

Sampling random networks

Phase 2:
- Use rewiring algorithm to remove all self and repeat loops.

Phase 3:
- Randomize network wiring by applying rewiring algorithm liberally.
- Rule of thumb: # Rewirings ≅ 10 × # edges\(^1\).

General random rewiring algorithm

- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.

- Rewire one end of each edge.
- Node degrees do not change.
- Works if \(e_1\) is a self-loop or repeated loop.
- Same as finding on/off/on/off 4-cycles. and rotating them.

Random sampling

- Problem with only joining up stubs is failure to randomly sample from all possible networks.
- Example from Milo et al. (2003)\(^1\):

![Diagram of network motifs]

1 configuration

90 configurations
Sampling random networks

- What if we have $P_k$ instead of $N_k$?
- Must now create nodes before start of the construction algorithm.
- Generate $N$ nodes by sampling from degree distribution $P_k$.
- Easy to do exactly numerically since $k$ is discrete.
- Note: not all $P_k$ will always give nodes that can be wired together.

Network motifs

- Looked at gene expression within full context of transcriptional regulation networks.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency $N_k$.
- Looked for certain subnetworks (motifs) that appeared more or less often than expected.

Network motifs

- Feedforward loop
  - $Z$ only turns on in response to sustained activity in $X$.
  - Turning of $X$ rapidly turns of $Z$.
  - Analogy to elevator doors.

Network motifs

- Single input module (SIM)
  - Master switch.
Network motifs

Reference I

On the uniform generation of random graphs with prescribed degree sequences, 2003. pdf

Network motifs in the transcriptional regulation network of *Escherichia coli*.

Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
For more, see work carried out by Wiggins et al. at Columbia.