

# Social Contagion

Principles of Complex Systems

CSYS/MATH 300, Spring, 2013 | #SpringPoCS2013

Prof. Peter Dodds  
@peterdodds

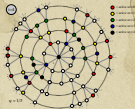
Department of Mathematics & Statistics | Center for Complex Systems |  
Vermont Advanced Computing Center | University of Vermont



Social Contagion  
Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



These slides brought to you by:

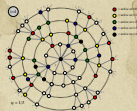
Social Contagion



## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Outline

## Social Contagion Models

Background

Granovetter's model

Network version

Final size

Spreading success

Groups

## References

## Social Contagion Models

Background

Granovetter's model

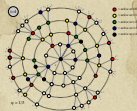
Network version

Final size

Spreading success

Groups

## References



# Things that spread well:

[buzzfeed.com](http://buzzfeed.com) (田):

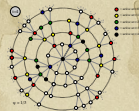


+ News ...

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



LOL + cute + fail + wtf:

Social Contagion

# Oopsie!



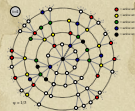
**BUZZFEED FELL DOWN AND WENT BOOM.**

Please try reloading this page. If the problem persists [let us know](#).

Social Contagion  
Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



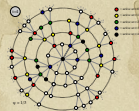
# The whole lolcats thing:



## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



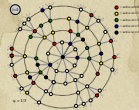
# Some things really stick:



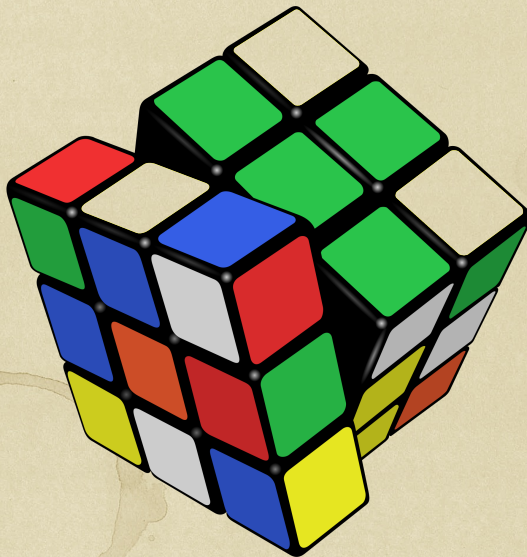
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



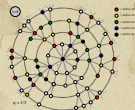
wtf + geeky + omg:



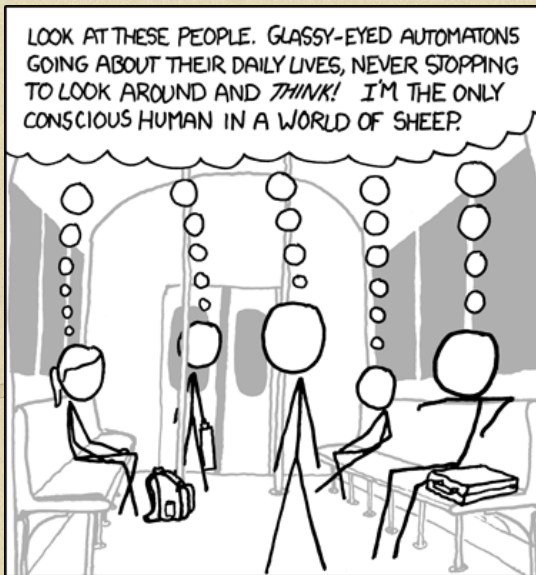
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References







<http://xkcd.com/610/> (田)

## Social Contagion Models

### Background

Granovetter's model

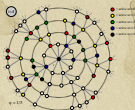
Network version

Final size

Spreading success

Groups

## References



# Social Contagion



## Social Contagion Models

### Background

Granovetter's model

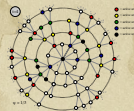
Network version

Final size

Spreading success

Groups

## References



# Social Contagion



## Social Contagion Models

### Background

Granovetter's model

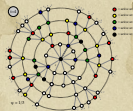
Network version

Final size


Spreading success

Groups

## References



## Examples abound

- ▶ fashion
- ▶ striking
- ▶ smoking (田) [7]
- ▶ residential segregation [19]
- ▶ ipods
- ▶ obesity (田) [6]
- ▶ Harry Potter
- ▶ voting
- ▶ gossip
- ▶ Rubik's cube 
- ▶ religious beliefs
- ▶ **leaving lectures**

## SIR and SIRS contagion possible

- ▶ Classes of behavior versus specific behavior: **dieting**

### Social Contagion Models

#### Background

Granovetter's model

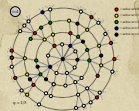
Network version

Final size

Spreading success

Groups

#### References



## Evolving network stories (Christakis and Fowler):

- ▶ The spread of quitting smoking (田) [7]
- ▶ The spread of spreading (田) [6]
- ▶ Also: happiness (田) [9], loneliness, ...
- ▶ The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (田)

## Controversy:

- ▶ Are your friends making you fat? (田) (Clive Thomson, NY Times, September 10, 2009).
- ▶ Everything is contagious (田)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Background

Granovetter's model

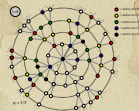
Network version

Final size

Spreading success

Groups

References



## Two focuses for us

- ▶ Widespread media influence
- ▶ Word-of-mouth influence

## We need to understand influence

- ▶ Who influences whom? Very hard to measure...
- ▶ What kinds of influence response functions are there?
- ▶ Are some individuals super influencers?  
Highly popularized by Gladwell<sup>[10]</sup> as 'connectors'
- ▶ The infectious idea of opinion leaders (Katz and Lazarsfeld)<sup>[16]</sup>

## Social Contagion Models

### Background

Granovetter's model

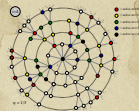
Network version

Final size

Spreading success

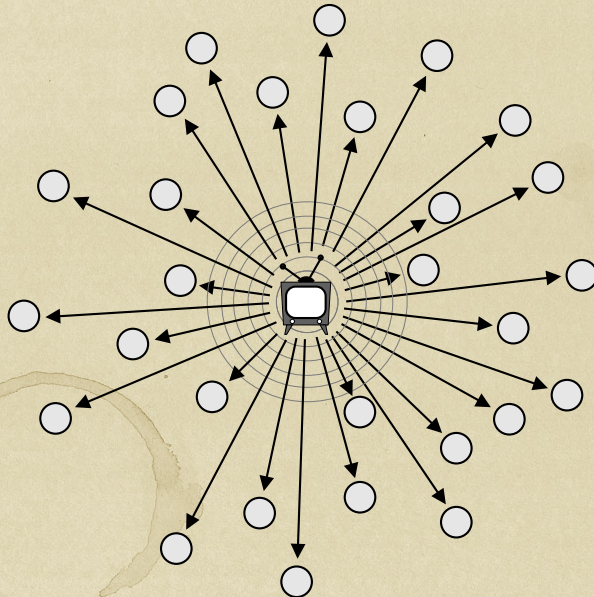
Groups

## References



# The hypodermic model of influence

Social Contagion



Social Contagion Models

Background

Granovetter's model

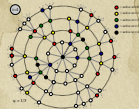
Network version

Final size

Spreading success

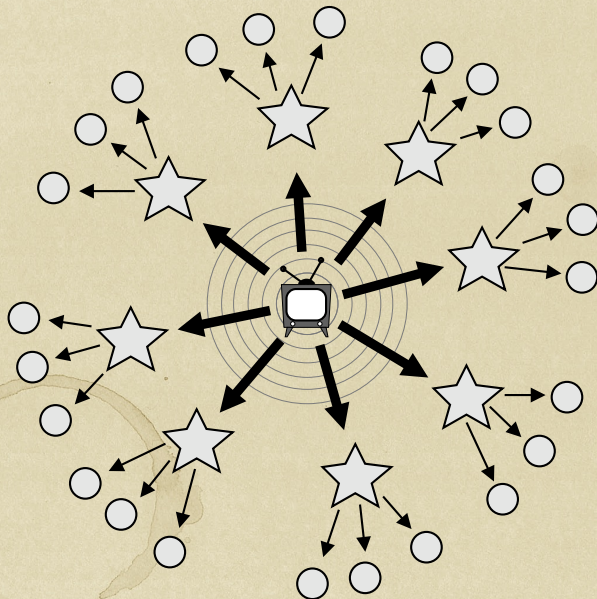
Groups

References



# The two step model of influence [16]

Social Contagion



## Social Contagion Models

### Background

Granovetter's model

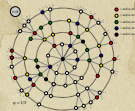
Network version

Final size

Spreading success

Groups

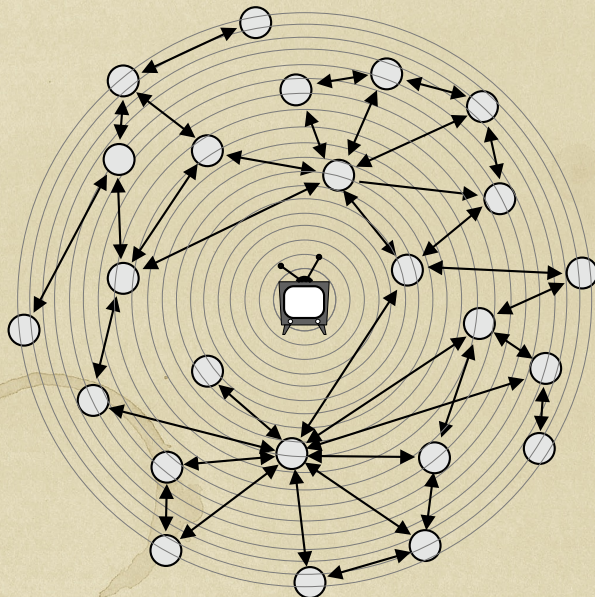
### References





# The general model of influence

Social Contagion



## Social Contagion Models

### Background

Granovetter's model

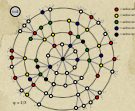
Network version

Final size

Spreading success

Groups

## References



## Why do things spread?

- ▶ Because of properties of special individuals?
- ▶ Or system level properties?
- ▶ Is the match that lights the fire important?
- ▶ Yes. But only because we are narrative-making machines...
- ▶ We like to think things happened for reasons...
- ▶ Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- ▶ System/group properties harder to understand
- ▶ Always good to examine what is said before and after the fact...

## Social Contagion Models

### Background

Granovetter's model

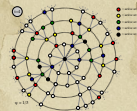
Network version

Final size

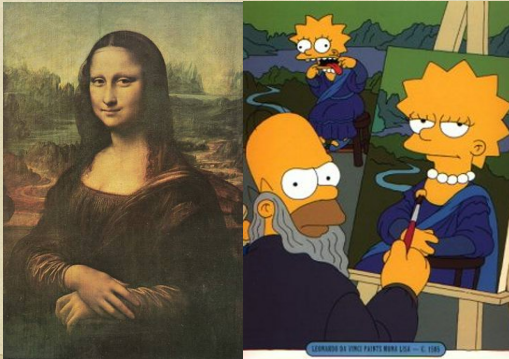
Spreading success

Groups

## References



# The Mona Lisa



- ▶ “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- ▶ Not the world’s greatest painting from the start...
- ▶ Escalation through theft, vandalism, **parody**, ...

## Social Contagion Models

### Background

Granovetter's model

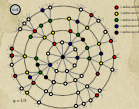
Network version

Final size

Spreading success

Groups

## References



# The completely unpredicted fall of Eastern Europe

Social Contagion



Social Contagion Models

Background

Granovetter's model

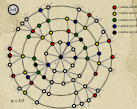
Network version

Final size

Spreading success

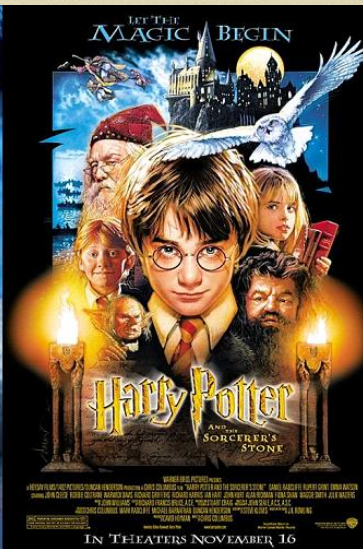
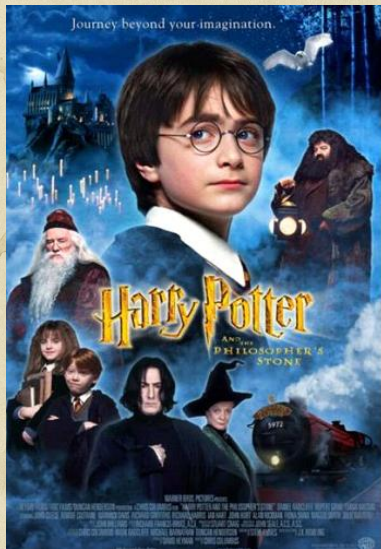
Groups

References



Timur Kuran: <sup>[17, 18]</sup> “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

# The dismal predictive powers of editors...



## Social Contagion Models

### Background

Granovetter's model

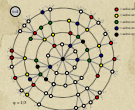
Network version

Final size

Spreading success

Groups

## References



## Messaging with social connections

- ▶ Ads based on message content (e.g., Google and email)
- ▶ BzzAgent (田)
- ▶ One of Facebook's early advertising attempts: Beacon (田)
- ▶ All of Facebook's advertising attempts.

## Social Contagion Models

### Background

Granovetter's model

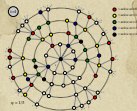
Network version

Final size

Spreading success

Groups

## References



# Getting others to do things for you

A very good book: 'Influence'<sup>[8]</sup> by Robert Cialdini (田)

## Six modes of influence:

1. **Reciprocation**: *The Old Give and Take... and Take*; e.g., Free samples, Hare Krishnas.
2. **Commitment and Consistency**: *Hobgoblins of the Mind*; e.g., Hazing.
3. **Social Proof**: *Truths Are Us*; e.g., Jonestown (田), Kitty Genovese (田) (contested).
4. **Liking**: *The Friendly Thief*; e.g., Separation into groups is enough to cause problems.
5. **Authority**: *Directed Deference*; e.g., Milgram's obedience to authority experiment. (田)
6. **Scarcity**: *The Rule of the Few*; e.g., Prohibition.

## Social Contagion Models

### Background

Granovetter's model

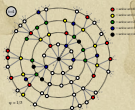
Network version

Final size

Spreading success

Groups

## References



- ▶ Cialdini's modes are heuristics that help up us get through life.
- ▶ Useful but can be leveraged...

## Other acts of influence:

- ▶ Conspicuous Consumption (Veblen, 1912)
- ▶ Conspicuous Destruction (Potlatch)

## Social Contagion Models

### Background

Granovetter's model

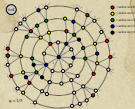
Network version

Final size

Spreading success

Groups

## References





## Some important models:

- ▶ Tipping models—Schelling (1971) [19, 20, 21]
  - ▶ Simulation on checker boards
  - ▶ Idea of thresholds
  - ▶ Explore the [Netlogo](#) (田) [online implementation](#) (田) [26]
- ▶ Threshold models—Granovetter (1978) [13]
- ▶ Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
  - ▶ Social learning theory, Informational cascades,...

## Social Contagion Models

### Background

Granovetter's model

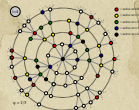
Network version

Final size

Spreading success

Groups

### References



## Thresholds

- ▶ Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- ▶ 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- ▶ Individual thresholds can vary
- ▶ Assumption: order of others' adoption does not matter... (**unrealistic**).
- ▶ Assumption: level of influence per person is uniform (**unrealistic**).

## Social Contagion Models

### Background

Granovetter's model

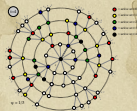
Network version

Final size

Spreading success

Groups

### References



## Some possible origins of thresholds:

- ▶ Inherent, evolution-devised inclination to coordinate, to conform, to imitate. <sup>[1]</sup>
- ▶ **Lack of information**: impute the worth of a good or behavior based on degree of adoption (social proof)
- ▶ Economics: **Network effects** or **network externalities**
  - ▶ Externalities = Effects on others not directly involved in a transaction
  - ▶ Examples: telephones, fax machine, Facebook, operating systems
  - ▶ An individual's utility increases with the adoption level among peers and the population in general

## Social Contagion Models

### Background

Granovetter's model

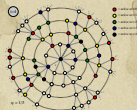
Network version

Final size

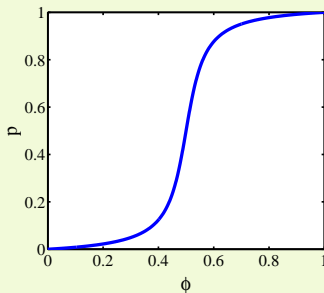
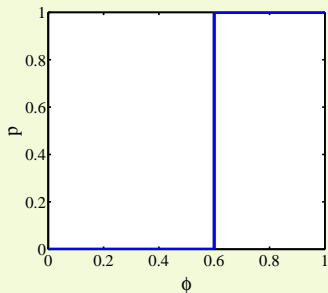
Spreading success

Groups

### References



# Threshold models—response functions



- ▶ Example threshold influence response functions:  
**deterministic** and **stochastic**
- ▶  $\phi$  = fraction of contacts 'on' (e.g., rioting)
- ▶ Two states: S and I.

## Social Contagion Models

Background

Granovetter's model

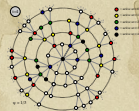
Network version

Final size

Spreading success

Groups

## References



## Granovetter's Threshold model—definitions

- ▶  $\phi^*$  = threshold of an individual.
- ▶  $f(\phi_*)$  = distribution of thresholds in a population.
- ▶  $F(\phi_*)$  = cumulative distribution =  $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)d\phi'_*$
- ▶  $\phi_t$  = fraction of people 'rioting' at time step  $t$ .

- ▶ At time  $t + 1$ , fraction rioting = fraction with  $\phi_* \leq \phi_t$ .

▶

$$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)d\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$

- ▶  $\Rightarrow$  Iterative maps of the unit interval  $[0, 1]$ .

## Social Contagion Models

Background

Granovetter's model

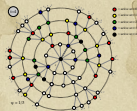
Network version

Final size

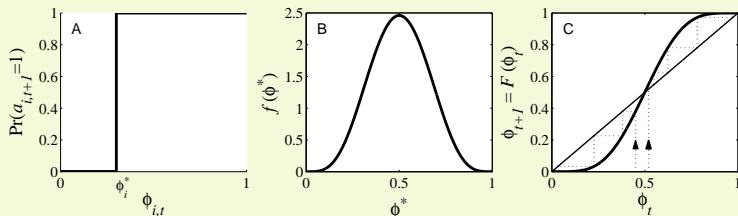
Spreading success

Groups

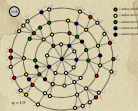
## References



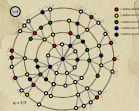
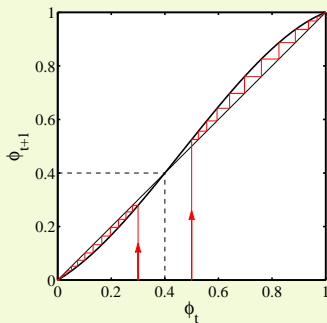
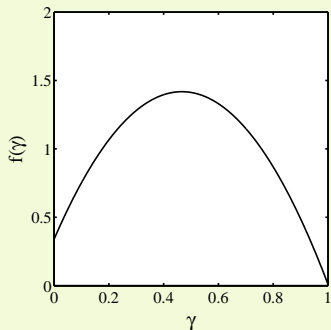
## Action based on perceived behavior of others:



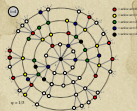
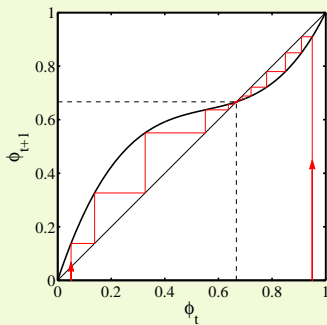
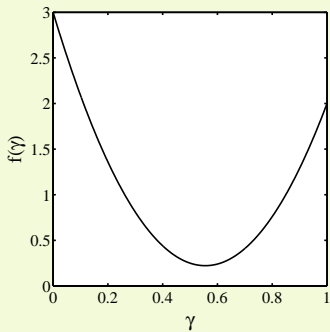
- ▶ Two states: S and I.
- ▶  $\phi$  = fraction of contacts 'on' (e.g., rioting)
- ▶ Discrete time update (strong assumption!)
- ▶ This is a **Critical mass model**



## Another example of critical mass model:

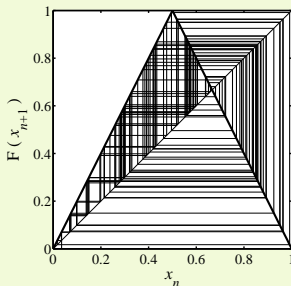
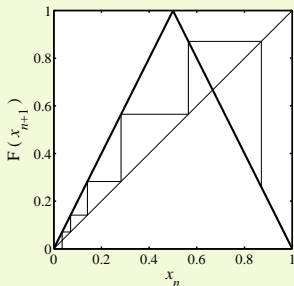


## Example of single stable state model:

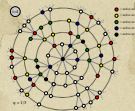




## Chaotic behavior possible [15, 14]



- ▶ Period doubling arises as map amplitude  $r$  is increased.
- ▶ Synchronous update assumption is crucial



# Threshold models—Nutshell

## Social Contagion Models

Background

**Granovetter's model**

Network version

Final size

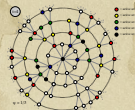
Spreading success

Groups

References

Implications for collective action theory:

1. Collective uniformity  $\nrightarrow$  individual uniformity
2. Small individual changes  $\Rightarrow$  large global changes

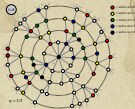


## “A simple model of global cascades on random networks”

- ▶ Many years after Granovetter and Soong’s work: D. J. Watts. Proc. Natl. Acad. Sci., 2002 [23]
  - ▶ Mean field model → network model
  - ▶ Individuals now have a limited view of the world

## We’ll also explore:

- ▶ “Seed size strongly affects cascades on random networks” [12]  
Gleeson and Cahalane, Phys. Rev. E, 2007.
- ▶ “Influentials, Networks, and Public Opinion Formation” [24]  
Watts and Dodds, J. Cons. Res., 2007.
- ▶ “Threshold models of Social Influence” [25]  
Watts and Dodds, The Oxford Handbook of Analytical Sociology, 2009.



# Threshold model on a network

- ▶ Interactions between individuals now represented by a network
- ▶ Network is **sparse**
- ▶ Individual  $i$  has  $k_i$  contacts
- ▶ Influence on each link is **reciprocal** and of **unit weight**
- ▶ Each individual  $i$  has a fixed threshold  $\phi_i$
- ▶ Individuals repeatedly poll contacts on network
- ▶ Synchronous, discrete time updating
- ▶ Individual  $i$  becomes active when fraction of active contacts  $\frac{a_i}{k_i} \geq \phi_i$
- ▶ Individuals remain active when switched (no recovery = SI model)

## Social Contagion Models

Background

Granovetter's model

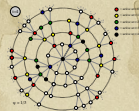
Network version

Final size

Spreading success

Groups

## References

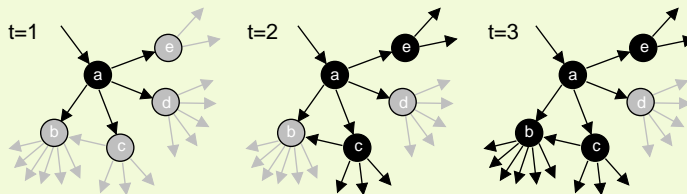


# Threshold model on a network

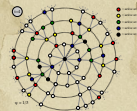
## Social Contagion Models

- Background
- Granovetter's model
- Network version**
- Final size
- Spreading success
- Groups

## References



► All nodes have threshold  $\phi = 0.2$ .



## First study random networks:

- ▶ Start with  $N$  nodes with a degree distribution  $p_k$
- ▶ Nodes are randomly connected (carefully so)
- ▶ Aim: Figure out when activation will propagate
- ▶ Determine a **cascade condition**

## The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

### Social Contagion Models

Background

Granovetter's model

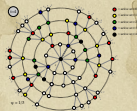
Network version

Final size

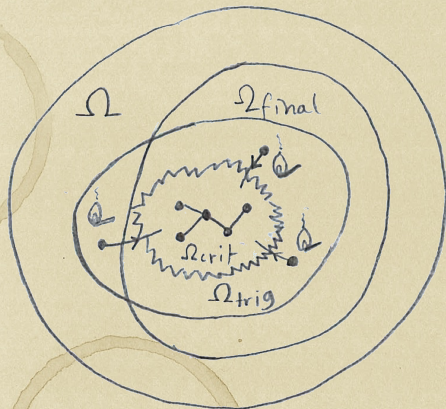
Spreading success

Groups

### References



# Example random network structure:



- ▶  $\Omega_{\text{crit}} = \Omega_{\text{vuln}} =$   
critical mass =  
global  
vulnerable  
component
- ▶  $\Omega_{\text{trig}} =$   
triggering  
component
- ▶  $\Omega_{\text{final}} =$   
potential extent  
of spread
- ▶  $\Omega =$  entire  
network

$$\Omega_{\text{crit}} \subset \Omega_{\text{trig}}; \Omega_{\text{crit}} \subset \Omega_{\text{final}}; \text{ and } \Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega.$$

## Social Contagion Models

Background

Granovetter's model

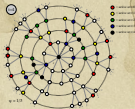
Network version

Final size

Spreading success

Groups

## References



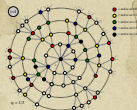
## Follow active links

- ▶ An active link is a link connected to an activated node.
- ▶ If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- ▶ We need to understand which nodes can be activated when only one of their neighbors becomes active.

## Social Contagion Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

## References





## Vulnerables:

- ▶ We call individuals who can be activated by just one contact being active **vulnerables**
- ▶ The vulnerability condition for node  $i$ :

$$1/k_i \geq \phi_i$$

- ▶ Which means # contacts  $k_i \leq \lfloor 1/\phi_i \rfloor$
- ▶ For global cascades on random networks, must have a *global cluster of vulnerables* [23]
- ▶ **Cluster of vulnerables = critical mass**
- ▶ Network story: 1 node  $\rightarrow$  critical mass  $\rightarrow$  everyone.

## Social Contagion Models

Background

Granovetter's model

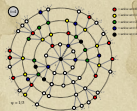
Network version

Final size

Spreading success

Groups

## References



## Back to following a link:

- ▶ A randomly chosen link, traversed in a random direction, leads to a degree  $k$  node with probability  $\propto kP_k$ .

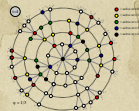
- ▶ Follows from there being  $k$  ways to connect to a node with degree  $k$ .

- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

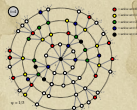


## Next: Vulnerability of linked node

- ▶ Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) d\phi'_*$$

- ▶ If linked node is **vulnerable**, it produces  **$k - 1$  new** outgoing active links
- ▶ If linked node is **not vulnerable**, it produces **no** active links.



## Putting things together:

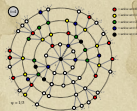
- ▶ Expected number of active edges produced by an active edge:

$$R = \sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1 - \beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}$$
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

## Social Contagion Models

[Background](#)[Granovetter's model](#)[Network version](#)[Final size](#)[Spreading success](#)[Groups](#)

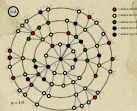
## References



So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶  $\beta_k$  = probability a degree  $k$  node is vulnerable.
- ▶  $P_k$  = probability a node has degree  $k$ .



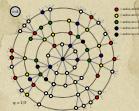
## Two special cases:

- ▶ (1) Simple disease-like spreading succeeds:  $\beta_k = \beta$

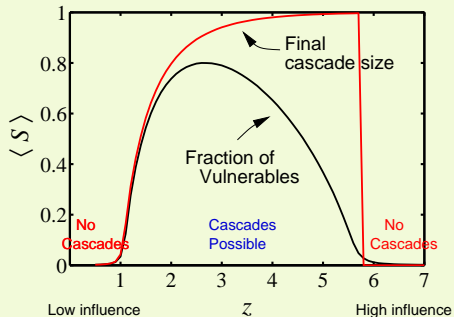
$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶ (2) Giant component exists:  $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$



# Cascades on random networks



Low influence  $z$  High influence



Example networks

- ▶ Cascades occur only if size of max vulnerable cluster  $> 0$ .
- ▶ System may be 'robust-yet-fragile'.
- ▶ 'Ignorance' facilitates spreading.

## Social Contagion Models

Background

Granovetter's model

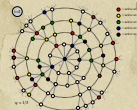
Network version

Final size

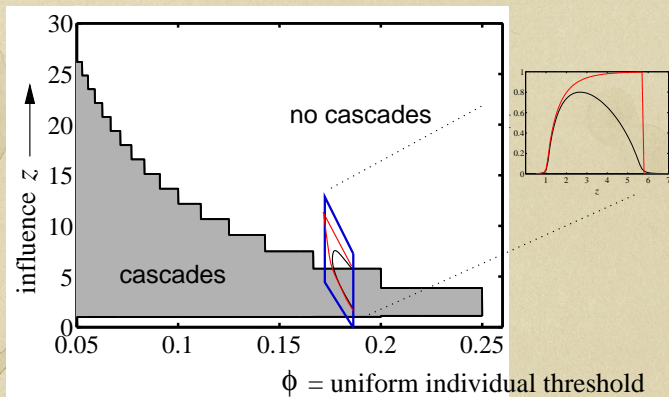
Spreading success

Groups

References



# Cascade window for random networks

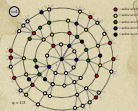


## Social Contagion Models

- Background
- Granovetter's model
- Network version**
- Final size
- Spreading success
- Groups

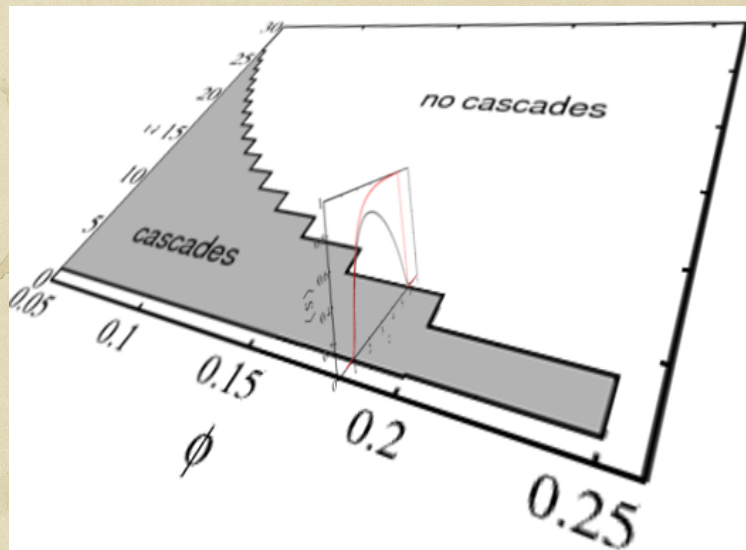
## References

- ▶ **'Cascade window'** widens as threshold  $\phi$  decreases.
- ▶ Lower thresholds enable spreading.





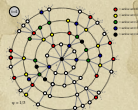
# Cascade window for random networks



## Social Contagion Models

- Background
- Granovetter's model
- Network version**
- Final size
- Spreading success
- Groups

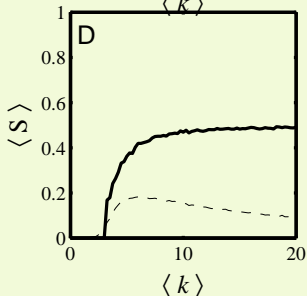
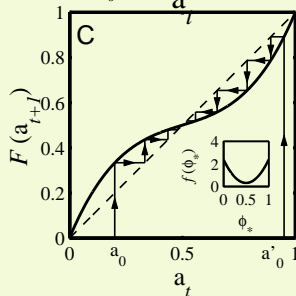
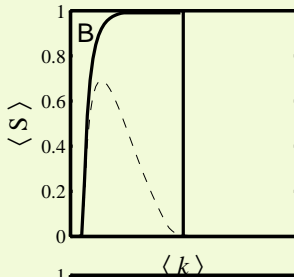
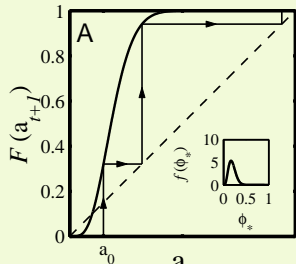
## References



# All-to-all versus random networks

all-to-all networks

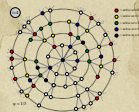
random networks



## Social Contagion Models

- Background
- Granovetter's model
- Network version**
- Final size
- Spreading success
- Groups

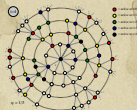
## References



# Cascade window—summary

For our simple model of a uniform threshold:

1. **Low**  $\langle k \rangle$ : No cascades in poorly connected networks. No global clusters of any kind.
2. **High**  $\langle k \rangle$ : Giant component exists but not enough vulnerables.
3. **Intermediate**  $\langle k \rangle$ : Global cluster of vulnerables exists. Cascades are possible in **“Cascade window.”**



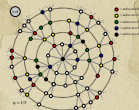
# Threshold contagion on random networks

- ▶ **Next:** Find expected fractional size of spread.
- ▶ Not obvious even for uniform threshold problem.
- ▶ Difficulty is in figuring out if and when nodes that need  $\geq 2$  hits switch on.
- ▶ Problem **solved** for infinite seed case by Gleeson and Cahalane:  
“Seed size strongly affects cascades on random networks,” Phys. Rev. E, 2007. <sup>[12]</sup>
- ▶ Developed further by Gleeson in “Cascades on correlated and modular random networks,” Phys. Rev. E, 2008. <sup>[11]</sup>

## Social Contagion Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

## References



# Expected size of spread

## Idea:

- ▶ Randomly turn on a fraction  $\phi_0$  of nodes at time  $t = 0$
- ▶ Capitalize on local branching network structure of random networks (again)
- ▶ Now think about what must happen for a specific node  $i$  to become active at time  $t$ :
  - $t = 0$ :  $i$  is one of the seeds (prob =  $\phi_0$ )
  - $t = 1$ :  $i$  was not a seed but enough of  $i$ 's friends switched on at time  $t = 0$  so that  $i$ 's threshold is now exceeded.
  - $t = 2$ : enough of  $i$ 's friends and friends-of-friends switched on at time  $t = 0$  so that  $i$ 's threshold is now exceeded.
  - $t = n$ : enough nodes within  $n$  hops of  $i$  switched on at  $t = 0$  and their effects have propagated to reach  $i$ .

## Social Contagion Models

Background

Granovetter's model

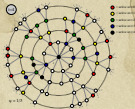
Network version

Final size

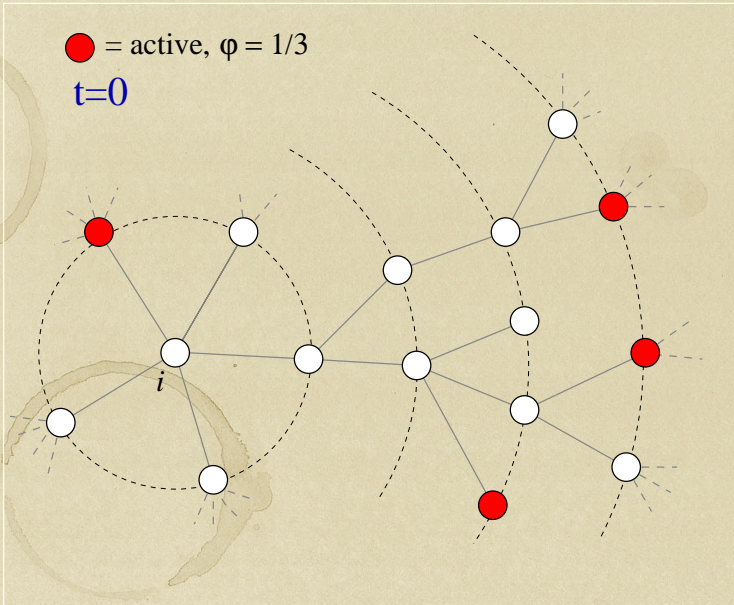
Spreading success

Groups

## References



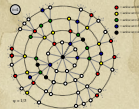
# Expected size of spread



## Social Contagion Models

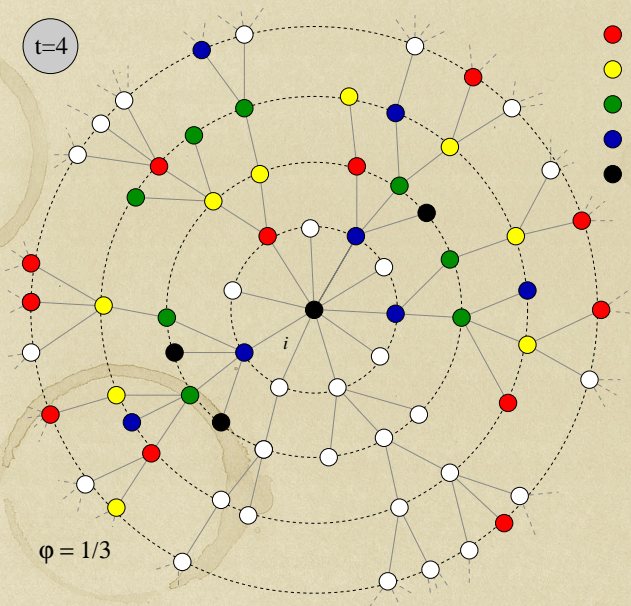
- Background
- Granovetter's model
- Network version
- Final size**
- Spreading success
- Groups

## References



# Expected size of spread

t=4

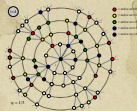


- = active at t=0
- = active at t=1
- = active at t=2
- = active at t=3
- = active at t=4

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



## Notes:

- ▶ Calculations are possible if nodes do not become inactive (strong restriction).
- ▶ Not just for threshold model—works for a wide range of contagion processes.
- ▶ We can analytically determine the entire time evolution, not just the final size.
- ▶ We can in fact determine  $\Pr(\text{node of degree } k \text{ switching on at time } t)$ .
- ▶ Asynchronous updating can be handled too.

## Social Contagion Models

Background

Granovetter's model

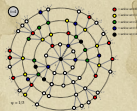
Network version

Final size

Spreading success

Groups

## References

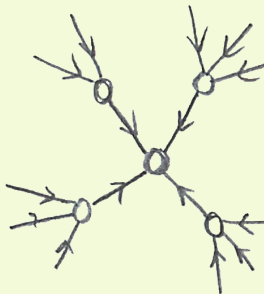
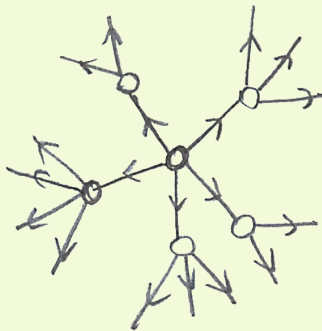




# Expected size of spread

## Pleasantness:

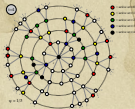
- ▶ Taking off from a single seed story is about **expansion** away from a node.
- ▶ Extent of spreading story is about **contraction** at a node.



## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Expected size of spread

► **Notation:**

$\phi_{k,t} = \mathbf{Pr}$ (a degree  $k$  node is active at time  $t$ ).

► **Notation:**  $B_{kj} = \mathbf{Pr}$  (a degree  $k$  node becomes active if  $j$  neighbors are active).

► Our starting point:  $\phi_{k,0} = \phi_0$ .

►  $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \mathbf{Pr}$  ( $j$  of a degree  $k$  node's neighbors were seeded at time  $t = 0$ ).

► Probability a degree  $k$  node was a seed at  $t = 0$  is  $\phi_0$  (as above).

► Probability a degree  $k$  node was not a seed at  $t = 0$  is  $(1 - \phi_0)$ .

► Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}.$$

## Social Contagion Models

Background

Granovetter's model

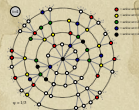
Network version

Final size

Spreading success

Groups

## References



# Expected size of spread

- ▶ For general  $t$ , we need to know the probability an edge coming into a degree  $k$  node at time  $t$  is active.
- ▶ **Notation:** call this probability  $\theta_t$ .
- ▶ We already know  $\theta_0 = \phi_0$ .
- ▶ Story analogous to  $t = 1$  case. For node  $i$ :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i j}.$$

- ▶ Average over all nodes to obtain expression for  $\phi_{t+1}$ :

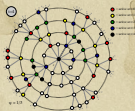
$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k j}.$$

- ▶ So we need to compute  $\theta_t$ ... massive excitement...

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Expected size of spread

First connect  $\theta_0$  to  $\theta_1$ :

▶  $\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{kj}$$

- ▶  $\frac{kP_k}{\langle k \rangle} = R_k = \mathbf{Pr}$  (edge connects to a degree  $k$  node).
- ▶  $\sum_{j=0}^{k-1}$  piece gives  $\mathbf{Pr}$ (degree node  $k$  activates) of its neighbors  $k - 1$  incoming neighbors are active.
- ▶  $\phi_0$  and  $(1 - \phi_0)$  terms account for state of node at time  $t = 0$ .
- ▶ See this all generalizes to give  $\theta_{t+1}$  in terms of  $\theta_t \dots$

## Social Contagion Models

Background

Granovetter's model

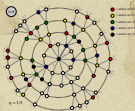
Network version

Final size

Spreading success

Groups

## References



# Expected size of spread

Two pieces: edges first, and then nodes

$$1. \theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

with  $\theta_0 = \phi_0$ .

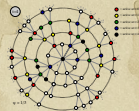
$$2. \phi_{t+1} =$$

$$\underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}}_{\text{social effects}}$$

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



## Expected size of spread:

- ▶ Retrieve cascade condition for spreading from a single seed in limit  $\phi_0 \rightarrow 0$ .
- ▶ Depends on map  $\theta_{t+1} = G(\theta_t; \phi_0)$ .
- ▶ First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning  $B_{k0} > 0$  for at least one value of  $k \geq 1$ .

- ▶ If  $\theta = 0$  is a fixed point of  $G$  (i.e.,  $G(0; \phi_0) = 0$ ) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k - 1) \bullet B_{k1} > 1.$$

### Social Contagion Models

Background

Granovetter's model

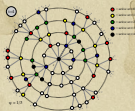
Network version

Final size

Spreading success

Groups

### References



# Expected size of spread:

## In words:

- ▶ If  $G(0; \phi_0) > 0$ , spreading must occur because some nodes turn on for free.
- ▶ If  $G$  has an **unstable fixed point** at  $\theta = 0$ , then cascades are also always possible.

## Non-vanishing seed case:

- ▶ Cascade condition is more complicated for  $\phi_0 > 0$ .
- ▶ If  $G$  has a **stable fixed point** at  $\theta = 0$ , and an **unstable fixed point** for some  $0 < \theta_* < 1$ , then for  $\theta_0 > \theta_*$ , spreading takes off.
- ▶ Tricky point:  $G$  depends on  $\phi_0$ , so as we change  $\phi_0$ , we also change  $G$ .

## Social Contagion Models

Background

Granovetter's model

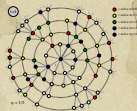
Network version

Final size

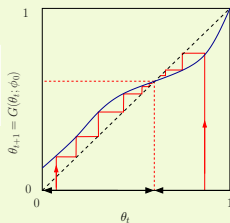
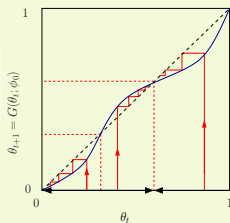
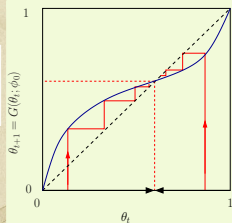
Spreading success

Groups

## References



# General fixed point story:

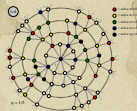


- ▶ Given  $\theta_0 (= \phi_0)$ ,  $\theta_\infty$  will be the nearest stable fixed point, either above or below.
- ▶ n.b., adjacent fixed points must have opposite stability types.
- ▶ **Important:** Actual form of  $G$  depends on  $\phi_0$ .
- ▶ So choice of  $\phi_0$  dictates both  $G$  and starting point—can't start anywhere for a given  $G$ .

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size**
- Spreading success
- Groups

## References



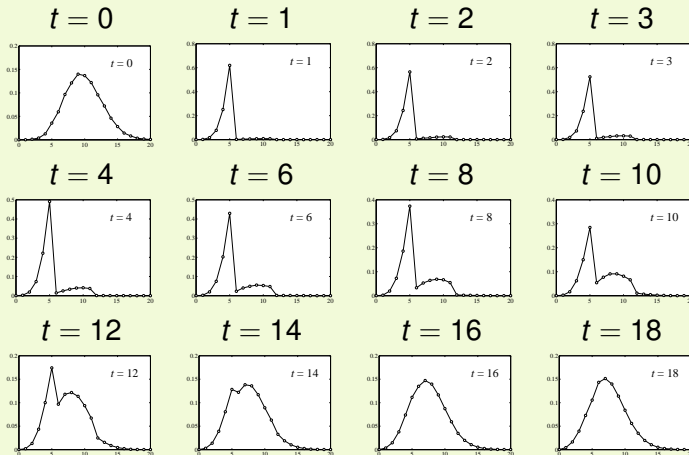


# Early adopters—degree distributions

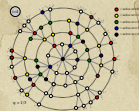
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success**
- Groups

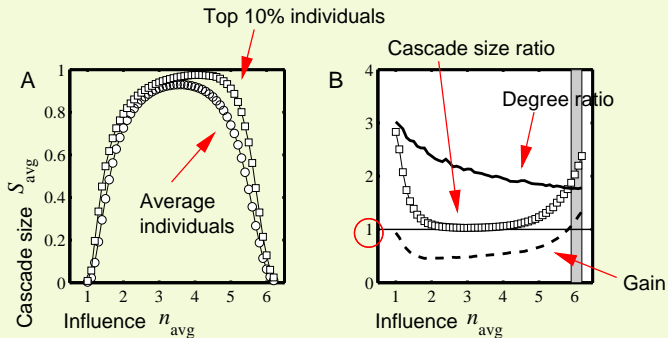
## References



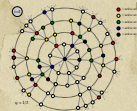
$P_{k,t}$  versus  $k$



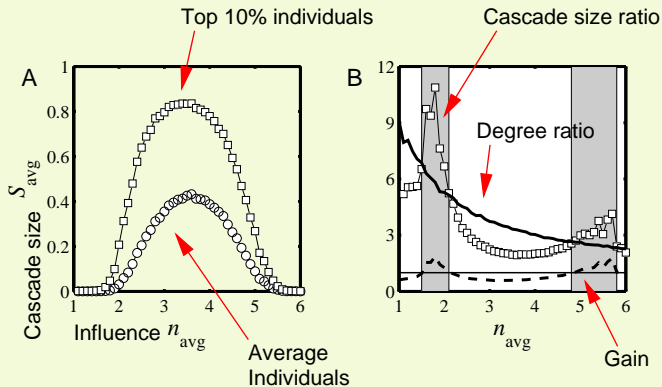
# The multiplier effect:



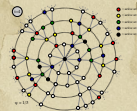
- ▶ Fairly uniform levels of individual influence.
- ▶ Multiplier effect is mostly below 1.



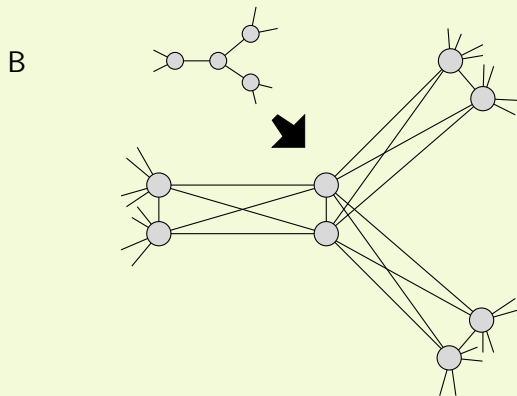
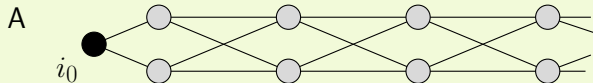
# The multiplier effect:



► Skewed influence distribution example.



# Special subnetworks can act as triggers

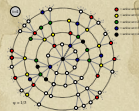


►  $\phi = 1/3$  for all nodes

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success**
- Groups

## References



# The power of groups...



## TEAMWORK

A FEW HARMLESS FLAKES WORKING TOGETHER CAN  
UNLEASH AN AVALANCHE OF DESTRUCTION.

[www.despair.com](http://www.despair.com)

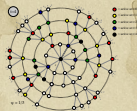
[despair.com](http://despair.com)

“A few harmless flakes  
working together can  
unleash an avalanche  
of destruction.”

### Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

### References

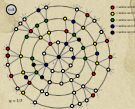


## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References

- ▶ Assumption of sparse interactions is good
- ▶ Degree distribution is (generally) key to a network's function
- ▶ Still, random networks don't represent all networks
- ▶ Major element missing: **group structure**

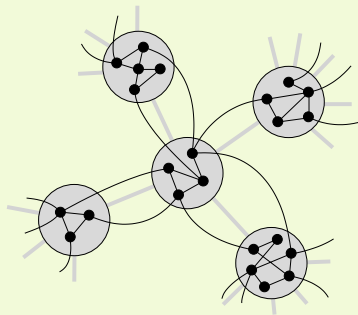


# Group structure—Ramified random networks

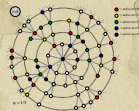
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

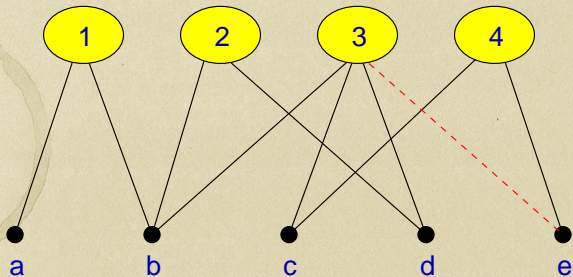
## References



$p$  = intergroup connection probability  
 $q$  = intragroup connection probability.

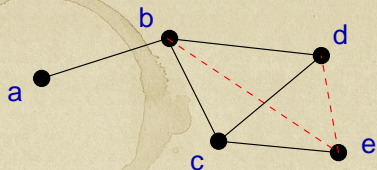


# Bipartite networks



[ contexts ]

[ individuals ]



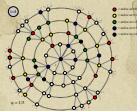
[ unipartite network ]

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success

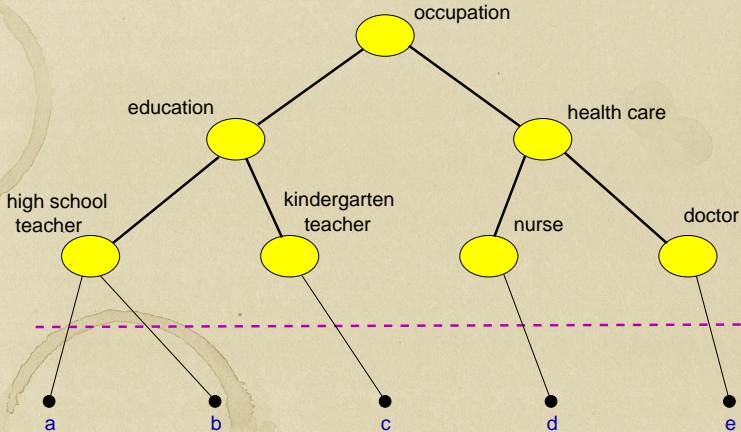
Groups

References





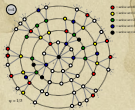
# Context distance



## Social Contagion Models

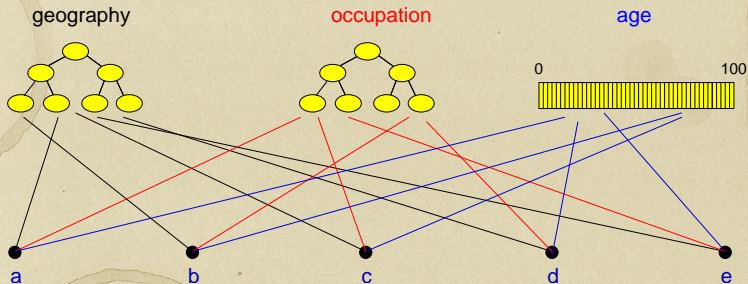
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Generalized affiliation model

Social Contagion



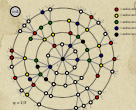
(Blau & Schwartz, Simmel, Breiger)

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success

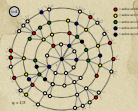
Groups

References



# Generalized affiliation model networks with triadic closure

- ▶ Connect nodes with probability  $\propto \exp^{-\alpha d}$   
where  
 $\alpha$  = homophily parameter  
and  
 $d$  = distance between nodes (height of lowest common ancestor)
- ▶  $\tau_1$  = intergroup probability of friend-of-friend connection
- ▶  $\tau_2$  = intragroup probability of friend-of-friend connection

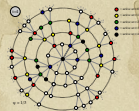
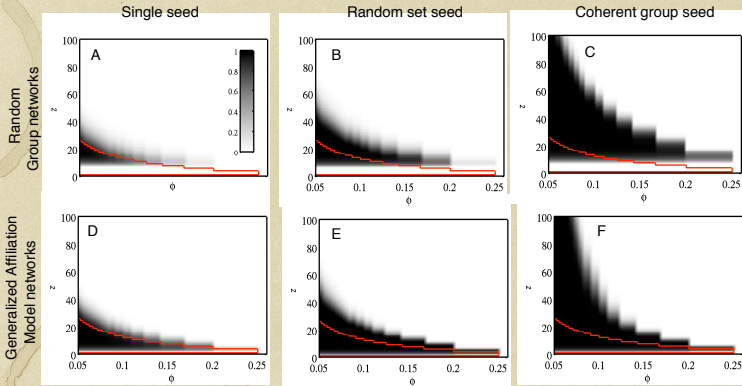


# Cascade windows for group-based networks

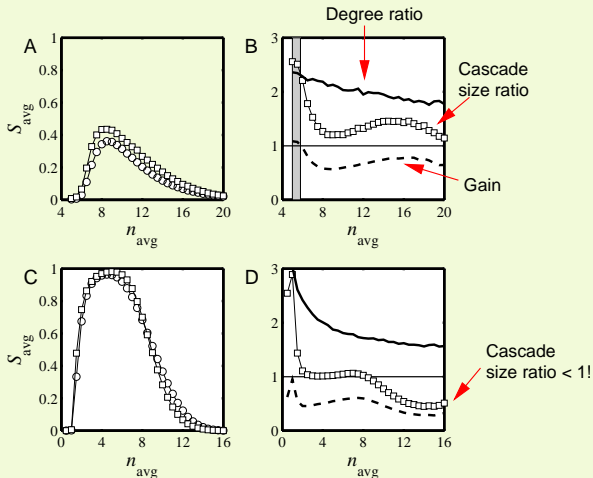
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Multiplier effect for group-based networks:

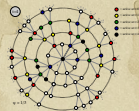


► Multiplier almost always below 1.

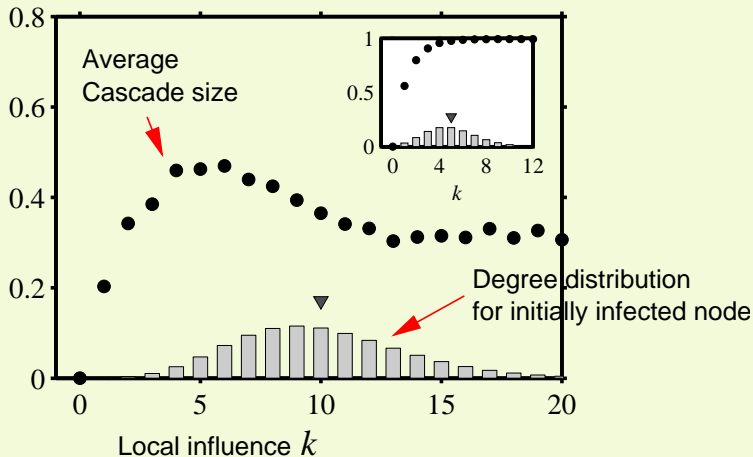
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



# Assortativity in group-based networks

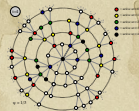


- ▶ The most connected nodes aren't always the most 'influential.'
- ▶ **Degree assortativity** is the reason.

## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References



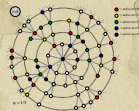
## Summary

- ▶ 'Influential vulnerables' are key to spread.
- ▶ Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- ▶ Groups may greatly facilitate spread.
- ▶ Seems that cascade condition is a global one.
- ▶ Most extreme/unexpected cascades occur in highly connected networks
- ▶ 'Influentials' are posterior constructs.
- ▶ Many potential influentials exist.

## Social Contagion Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success

## References



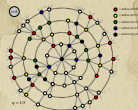
## Implications

- ▶ Focus on **the influential vulnerables**.
- ▶ Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- ▶ Only **simple ideas** can spread by word-of-mouth.  
(Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
- ▶ Displaying can be **passive** = free (yo-yo's, fashion), or **active** = harder to achieve (political messages).
- ▶ Entities can be novel or designed to combine with others, e.g. block another one.

## Social Contagion Models

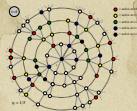
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

## References





- [1] A. Bentley, M. Earls, and M. J. O'Brien.  
I'll Have What She's Having: Mapping Social Behavior.  
MIT Press, Cambridge, MA, 2011.
- [2] S. Bikhchandani, D. Hirshleifer, and I. Welch.  
A theory of fads, fashion, custom, and cultural change as informational cascades.  
J. Polit. Econ., 100:992–1026, 1992.
- [3] S. Bikhchandani, D. Hirshleifer, and I. Welch.  
Learning from the behavior of others: Conformity, fads, and informational cascades.  
J. Econ. Perspect., 12(3):151–170, 1998. [pdf](#) (☐)



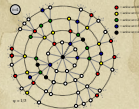
# References II

- [4] J. M. Carlson and J. Doyle.  
Highly optimized tolerance: A mechanism for power laws in designed systems.  
[Phys. Rev. E, 60\(2\):1412–1427, 1999. pdf](#) (田)
- [5] J. M. Carlson and J. Doyle.  
Highly optimized tolerance: Robustness and design in complex systems.  
[Phys. Rev. Lett., 84\(11\):2529–2532, 2000. pdf](#) (田)
- [6] N. A. Christakis and J. H. Fowler.  
The spread of obesity in a large social network over 32 years.  
[New England Journal of Medicine, 357:370–379, 2007. pdf](#) (田)

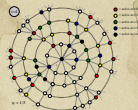
## Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

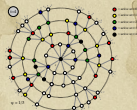
## References



- [7] N. A. Christakis and J. H. Fowler.  
The collective dynamics of smoking in a large social network.  
[New England Journal of Medicine, 358:2249–2258, 2008. pdf \(田\)](#)
- [8] R. B. Cialdini.  
[Influence: Science and Practice.](#)  
Allyn and Bacon, Boston, MA, 4th edition, 2000.
- [9] J. H. Fowler and N. A. Christakis.  
Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.  
[BMJ, 337:article #2338, 2008. pdf \(田\)](#)



- [10] M. Gladwell.  
The Tipping Point.  
Little, Brown and Company, New York, 2000.
- [11] J. P. Gleeson.  
Cascades on correlated and modular random networks.  
[Phys. Rev. E, 77:046117, 2008. pdf](#) (田)
- [12] J. P. Gleeson and D. J. Cahalane.  
Seed size strongly affects cascades on random networks.  
[Phys. Rev. E, 75:056103, 2007. pdf](#) (田)
- [13] M. Granovetter.  
Threshold models of collective behavior.  
[Am. J. Sociol., 83\(6\):1420–1443, 1978. pdf](#) (田)



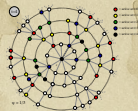
# References V

- [14] M. Granovetter and R. Soong.  
Threshold models of diversity: Chinese restaurants,  
residential segregation, and the spiral of silence.  
[Sociological Methodology, 18:69–104, 1988. pdf](#) (田)
- [15] M. S. Granovetter and R. Soong.  
Threshold models of interpersonal effects in  
consumer demand.  
[J. Econ. Behav. Organ., 7:83–99, 1986. pdf](#) (田)
- [16] E. Katz and P. F. Lazarsfeld.  
Personal Influence.  
The Free Press, New York, 1955.
- [17] T. Kuran.  
Now out of never: The element of surprise in the  
east european revolution of 1989.  
[World Politics, 44:7–48, 1991. pdf](#) (田)

## Social Contagion Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

## References

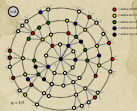


- [18] T. Kuran.  
Private Truths, Public Lies: The Social Consequences of Preference Falsification.  
Harvard University Press, Cambridge, MA, Reprint edition, 1997.
- [19] T. C. Schelling.  
Dynamic models of segregation.  
J. Math. Sociol., 1:143–186, 1971. pdf (⊞)
- [20] T. C. Schelling.  
Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.  
J. Conflict Resolut., 17:381–428, 1973. pdf (⊞)
- [21] T. C. Schelling.  
Micromotives and Macrobehavior.  
Norton, New York, 1978.

Social Contagion  
Models

Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



[22] D. Sornette.

Critical Phenomena in Natural Sciences.  
Springer-Verlag, Berlin, 1st edition, 2003.

[23] D. J. Watts.

A simple model of global cascades on random networks.

Proc. Natl. Acad. Sci., 99(9):5766–5771, 2002.

[pdf](#) (田)

[24] D. J. Watts and P. S. Dodds.

Influentials, networks, and public opinion formation.

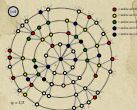
Journal of Consumer Research, 34:441–458, 2007.

[pdf](#) (田)

Social Contagion  
Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



[25] D. J. Watts and P. S. Dodds.  
Threshold models of social influence.  
In P. Hedström and P. Bearman, editors, The Oxford Handbook of Analytical Sociology, chapter 20, pages 475–497. Oxford University Press, Oxford, UK, 2009. pdf (田)

[26] U. Wilensky.  
Netlogo segregation model.  
<http://ccl.northwestern.edu/netlogo/models/Segregation>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL., 1998.

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

