Chapter 37

SOME FUN, THIRTY-FIVE YEARS AGO

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Abstract

A pencil-and-paper experiment with spacial segregation leads to some general phenomena of spatial organization.

Keywords

segregation, integration, neighbor, neighborhood, majority, minority

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Sometime in the 1960s I wanted to teach my classes how people's interactions could lead to results that were neither intended nor expected. I had in mind associations or spatial patterns reflecting preferences about whom to associate with in neighborhoods, clubs, classes, or ballparks, or at dining tables. Whether racial or linguistic differences or differences in age or income and wealth were what I had in mind, I'm not sure now. I spent a summer at RAND and took advantage of RAND's library to thumb through a few decades of sociological journals, looking for illustrative material that I could assign to my students. I found nothing I could use, and decided I'd have to work something out for myself.

One afternoon, settling into an airplane seat, I had nothing to read. To amuse myself I experimented with pencil and paper. I made a line of x's and o's that I somehow randomized, and postulated that every x wanted at least half its neighbors to be x's and similarly with o's. Those that weren't satisfied would move to where they were satisfied. This was tedious because I had no eraser, but I persuaded myself that the results could prove interesting.

At home I took advantage of my son's coin collection. He had quantities of pennies, both copper and the gray zinc one's we had all used during the war. I spread them out in a line, either in random order or any haphazard way, gave the coppers and the zincs their own preferences about neighbors, and moved the discontents—starting at the left and moving steadily to the right—to where they might inject themselves between two others in the line and be content. The results astonished me. But as I reflected, and as I experimented, the results became plausible and ultimately obvious.

Just to remind you, a line of randomly distributed coppers and zincs that looks like this,

0+000++0+00++00+++0++0++00++00++00++0+0+00+++0++00000+++000+00++
0+0++0,

when each wants at least four out of the eight nearest neighbors to be one's own type, becomes after two "rounds" of moving:

00000000+++++++++++++++0000000000+++++++++++++++0000000000000000
++++++.

I experimented with different sizes of "neighborhoods"—the six, eight, or ten surrounding coins, different preferences—half like oneself, one-quarter like oneself, and different majority–minority ratios, and got results that fascinated me.

A one-dimensional line couldn't take me very far. But in two dimensions it wasn't clear how to intrude a copper or a zinc into the midst of coppers and zincs. I mentioned this problem to Herb Scarf, who suggested I put my pennies on a checkerboard leaving enough blank spaces to make search and satisfaction possible.

So I made a 16 × 16 checkerboard, located zincs and coppers at random with about a fifth of the spaces blank, got my twelve-year-old to sit across from me at the coffee table, and moved discontented zincs and coppers to where their demands for like or unlike neighbors were met. We quickly found out it didn't matter much in what order we selected the discontents to move—from middle outward, from out inward, from left to right or diagonally. We kept getting the same kind of results. The dynamics were sufficiently intriguing to keep my twelve-year-old engaged.

I found things I hadn't expected. Usually, once found, they appeared obvious. If zincs and coppers were majority and minority, or if zincs and coppers had greater and lesser demands for like neighbors, the sizes of eventual clusters and the densities of the different clusters varied accordingly.

And when we postulated that zincs and coppers had positive desires for unlike neighbors, especially if they were minority and majority, we got results that appeared weird until we saw what was happening. (The minority, desired as neighbors, had to become "rationed" among the majority.)

I had an interesting experience with computers at that time. I knew nothing about what computers could do, or how they did it, but I knew that RAND had people who did. I approached RAND and asked to be in touch with somebody who could program what I'd been doing. Somebody was put in touch with me. I quickly learned something crucial: programmer and experimenter must work closely, the former understanding what the latter wants, the latter understanding how programs work. Three thousand miles apart we didn't work that way. For me the results were perplexing. I eventually caught on that I had individuals counting themselves as their own "neighbors", had individuals on edges of the board or in corners miscounting how many neighbors they had, and in other ways had inadequately stipulated exactly how the zincs and coppers were to respond.

I later got James Vaupel to program things in Basic, but he was about to leave for the summer and I needed to know how to reprogram myself. We met on a Sunday, with sandwiches and beer, and in about five hours he taught me how to program with whatever parameters I wanted. He left the next day, but I was prepared.

Incidentally, the person at RAND who did the programming for me was John Casti. Thirty years later—I had never met him—he mentioned, in the course of a presentation that I attended, that his experience with my neighborhood patterning had initiated him into a career in simulation.

I published, along with the "checkerboard" model, a purely analytical model that I called the "bounded neighborhood" model [Schelling (1971)]. That model postulated a finite location that a person was either in or not in, positions within the neighborhood not being of concern. (It could be a model of membership or enrollment or participation, not necessarily location.) I thought the results I got from that model were as interesting as those from the checkerboard, but nobody else appeared to think so. I also explored the nature of a collective "tipping point" in a chapter in Tony Pascal's book, published about a year later, with a purely analytical model [Pascal (1972)]. It got little attention. In that "bounded neighborhood" model it became clear that an important phenomenon can be that a too-tolerant majority can overwhelm a minority and bring about segregation.

I've never been sure why my little simulation got so much attention after so many years. I discovered twenty-five years later that I'd been some kind of pioneer. It must be some limitation of my scientific imagination that I'd no idea I was doing something generic, something with promise beyond my neighborhood application.

I've had one experience that others may have had, in publishing a much abbreviated version of that model in a book [Schelling (1978)], believing that the full treatment in the Journal of Mathematical Sociology [Schelling (1971)] might be more than readers of the book would need. References to my model are usually to the version in the book, not to the original. I've seen no reference, for example, to the results I got when I postulated a strong preference for neighbors of opposite type. If one is interested in the "neighborhood" effects of differences other than in color or race, especially with individuals of one type much scarcer than individuals of the other type, the "integrationist" preferences become highly plausible. (I put "neighborhood" in quotation marks because residence is not the only interpretation.)

Another interesting result in the original, but not in the book version, a result that somewhat surprised me until I saw how it worked—an advantage of doing it manually instead of on a computer—is that if one subjects all the actors to a fairly strict limit on movement the results are usually that everyone becomes satisfied with less travel and more integration. For example—the linear case is adequate to illustrate—if we impose on all the +'s and 0's a restriction that no one may move more than five spaces, moving to the best available position if satisfaction cannot be achieved within five spaces, the original random line we used above becomes, in one round,

00000++++++000000+++++++++000000++++++00000+++++0000000000+++++++
++000,

All except two of the three on the right are satisfied, on average individuals traveled less than half the distance, and this is much more "integrated." The total number of unlike neighbors in this "restricted travel" version is twice that of the original equilibrium. And in the original, 30 of the 70 individuals ended up with no neighbors at all of opposite type; in this case of restricted movement, only 5.

This restricted-movement example is one of several results that may be unanticipated but become obvious with a little experience. Analytically one might say that restricting movement is a substitute for collaboration or anticipation. Unrestricted—and in the absence of collaboration or anticipation—an individual 0 will move to the nearest cluster of 4 or more 0's, passing numerous lonely 0's in what may be a long journey. Sufficiently restricted, the lonely 0 may be able only to join the nearest lonely 0, far from satisfactory; but the next lonely 0 looking for company can now join the two, making it three, and shortly a fourth will arrive and a fifth. (Increasing the "price" of travel may reduce the "cost" of travel.) By moving, individuals both add and subtract externalities where they leave, and add and subtract where they settle.

A similar principle is observed if the 0's are a minority and the +'s a majority. I remember being so confident that the smaller the minority relative to the majority, the smaller would be the minority clusters, that I wrote that before I tried it. When I tried it, it didn't work; the opposite occurred: the minority clusters became absolutely larger as the minority itself became smaller. What I had originally thought to be so obvious I needn't bother to demonstrate it turned out, upon demonstration, to be just as obviously the opposite.

Now that computers can display all the movement in "real time" there is, I suppose, little advantage in doing this kind of thing manually, but when I was doing it computers could compute but not display, and I often got computer results I could make little sense of until I worked it by hand.

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