

# An asymmetrical model of the airways of the dog lung

KEITH HORSFIELD, WENDY KEMP, AND SALLY PHILLIPS

*The Midhurst Medical Research Institute, Midhurst, West Sussex, England GU29 0BL*

HORSFIELD, KEITH, WENDY KEMP, AND SALLY PHILLIPS. *An asymmetrical model of the airways of the dog lung*. *J. Appl. Physiol.: Respirat. Environ. Exercise Physiol.* 52(1): 21-26, 1982.—A resin cast of the bronchial tree of a dog was studied down to branches of 0.5-mm diameter. The branches were ordered, and the number of branches, mean diameter, and mean length of branches in each order were determined. The model was developed from these data with the property that delta, the difference in order between the two daughter branches at a bifurcation, is identical for all parent branches of a given order. This property facilitates the use of the model for the calculation of physiological variables such as input impedance (Fredberg and Hoenig, *ASME J. Biomech. Eng.* 100: 57-66, 1978), and the comparison of such results with those obtained from symmetrical models (Sidell and Fredberg, *ASME J. Biomech. Eng.* 100: 131-138, 1978).

bronchial tree morphology; orders of branching; airway dimensions

IT HAS BEEN KNOWN FOR MANY YEARS that the mammalian bronchial tree branches asymmetrically, but the first study describing the distribution of this asymmetry in the dog lung was by Ross (9). The symmetrical human bronchial tree model A of Weibel (14) is well known, but his asymmetrical model B has been little used. Another investigation of the human bronchial tree, with emphasis on asymmetry, was made by Horsfield and Cumming (5). However, in none of these studies of asymmetrical lungs are the data easy to use for physiological calculations because the "connectivity," i.e., the pattern in which the branches join each other, is undefined. The first asymmetrical model of the bronchial tree to include connectivity was described by Horsfield et al. (7), and this paved the way for some more detailed analyses of the effects of asymmetry in the human lung (1, 10, 16). Detailed data from six mammalian lungs of four species, including dimensions and connectivity, were given by Raabe et al. (8); these data are in a form suitable for computer analysis. The purpose of the present paper is to develop a model of the airways of the dog lung after the style of Fredberg and Hoenig (1), an approach that facilitates computer analysis.

## METHODS

The data on which the model is based were obtained from a resin cast of the bronchial tree of a 25-kg dog. The method of making such casts has been described

previously (6). The lungs were fixed at an inflation pressure of 25 cmH<sub>2</sub>O, and the polyester resin (AP 101 PA, Trylon, Wellingborough, Northants, England) was then injected at a pressure of 30 cm resin. To facilitate study the cast was divided into two zones, a proximal zone down to branches of 2.0-mm diameter, and a distal zone from 2.0- to 0.5-mm diameter. Seven distal zone samples, one from each lobe, were broken off the cast at bronchi about 2.0 mm in diameter. They were studied under a binocular dissecting microscope at a magnification of  $\times 10$  or  $\times 20$ . Dimensions were measured with a scale in one eyepiece, to the nearest 0.025 mm, equivalent to half a division on the scale at  $\times 20$ . Diameters were measured at the midpoint of a branch between two bifurcations, and lengths from the carina or apex of one branching angle to the next along a line parallel with the center line of the branch. Each sample was drawn, and the drawing was colored to indicate orders, an order 1 branch being defined as the first branch up any pathway equal to or greater than 0.5-mm diameter; subsequent orders follow automatically once the order 1 branches have been defined. The method of ordering used was that described previously as "divisions up" (5), a method by which order increases by one at each bifurcation up from the periphery towards the trachea (Fig. 1). It was chosen in preference to Strahler's method (12, 13) because it was necessary to model the branching segments between all the points of bifurcation, which Strahler's method does not do. A comparison between these two methods of ordering has been published previously (2).

Study of the samples showed that the cutoff at 0.5-mm diameter was at about the level of the terminal bronchioles. This was judged by the appearance of alveolar markings on the respiratory bronchioles distal to them. An exact definition of a terminal bronchiole in anatomical terms is in fact remarkably difficult, and the diameter of 0.5 mm for this structure must be regarded as only approximate.

After the peripheral zone samples had been broken off, the main cast was pruned to branches of 2.0-mm diameter or slightly less, and all of the proximal zone branches were counted and measured. An order 1 branch in the proximal zone was defined as the first branch up any pathway equal to or greater than 2.0-mm diameter. Each branch on the cast was painted according to a color code to indicate its order. Dimensions were measured with pointed calipers on a millimeter scale, judged by eye to the nearest 0.25 mm.

When the ordering and measuring of branches in each zone were completed, the number in each order was counted, and their mean diameter and length calculated. The order of a broken branch was determined by attributing to it the average order of intact branches of similar diameter. Numbers of branches missing distal to a broken branch were calculated by a new iterative procedure. When the number of branches in each order is plotted on a logarithmic scale against order, a linear relation is found. Thus the number of branches in successive orders increases by a constant factor called the branching ratio ( $R_b$ ). For the corrections  $R_b$  was set at an initial value of 1.3, and for each broken order 2 branch 1.3 order 1 branches were allowed. Similarly, for each broken order 3 branch, 1.3 order 2 branches and  $1.3 \times 1.3$  order 1 branches were allowed. The same procedure was followed for higher order broken branches. Next, the branching ratio was calculated from the antilog of the line of least-squares best fit to the corrected data, and the new value of  $R_b$  was used to recalculate the corrections. This procedure was repeated until  $R_b$  converged to a constant value, which occurred after only two iterations. Finally the corrected numbers were rounded to the nearest integers.

Branching in the dog bronchial tree appears to be predominantly monopodial, at least in the proximal zone. This means that there are one or two axial branches in a lobe that continue in a relatively straight line with branches coming off laterally (Fig. 2A). Even when branches arise close together there is no great problem in ordering them (Fig. 2B). When two branches arise at the same level (Fig. 2C) an order is still attributed, even though it might have zero length. This approach was adopted by Raabe et al. (8) to deal with apparent trichotomies.

## RESULTS

Table 1 shows the total number of branches and the corrections for broken branches in the seven distal zone samples. Figures 3 and 4 show the number, mean diameter, and mean length of branches in each order in the distal and proximal zones, respectively. It was observed that the data for number of branches could be fitted by three straight lines on a logarithm of number against

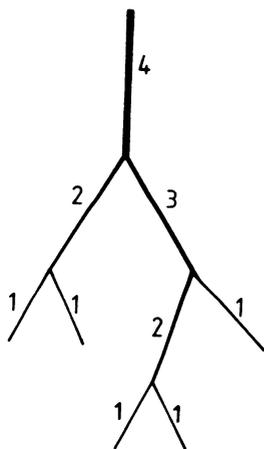


FIG. 1. A branching tree showing orders.

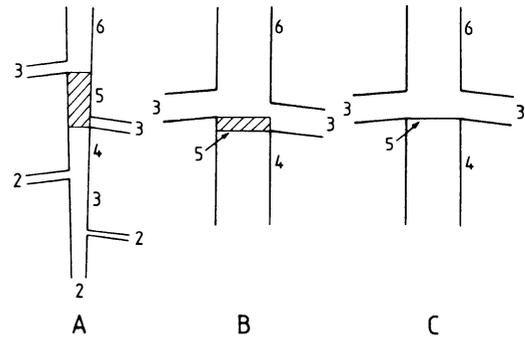


FIG. 2. A: part of a monopodial branching tree with orders showing a straightforward order 5 branch. B: short order 5 branch where two lateral branches arise close together. C: zero-length order 5 branch where two lateral branches arise at same level.

TABLE 1. Corrections for broken branches in pooled data from the seven distal zone samples

Order	Intact Branches	Broken Branches	Corrections Added On	Corrected Total
1	108		23	131
2	65	1	16	82
3	48	2	10	60
4	34	1	7	42
5	27		4	31
6	19		4	23
7	15	1	2	18
8	11		1	12
9	10	1		11
10	8			8
11	6			6
12	5			5
13	1			1
14	1			1
15	1			1

The numbers of order-1 branches in the seven samples were, respectively, 14, 15, 15, 12, 22, 15, and 15. On only one of them were orders 13-15 represented.

order plot, determined by a least-squares best fit. The lines were for orders 6-25 in the proximal zone, 1-6 in the proximal zone, and 1-12 in the distal zone. In the top three orders of each zone there was only one branch of each order. This was because of the very asymmetrical, or monopodial, pattern of branching. Because of the small numbers, these orders were excluded from the calculation of the branching ratios. Table 2 gives the  $R_b$  obtained from each line; it will be seen that the distal ends of the proximal zone and the distal zone have similar values of 1.330 and 1.338, respectively. This suggests that orders 12-15 of the distal zone most probably have an  $R_b$  somewhere between 1.330 and 1.338. We have arbitrarily chosen the lower figure to estimate their numbers. Thus order 1 of the proximal zone has 187 branches, and  $187 \times 1.33$  gives an estimated 249 branches in order 15 of the distal zone. Similarly, for orders 14, 13, and 12 respectively,  $249 \times 1.33 = 331$ ,  $331 \times 1.33 = 440$ , and  $440 \times 1.33 = 585$ .

The bronchi sampled from the distal zone represent a proportion of the total population of that zone. Although we can never be certain that the sample is truly representative, we assume that it is where reasonable numbers have been obtained. This was not the case with orders 13-15 as there was only one of each, so the numbers of

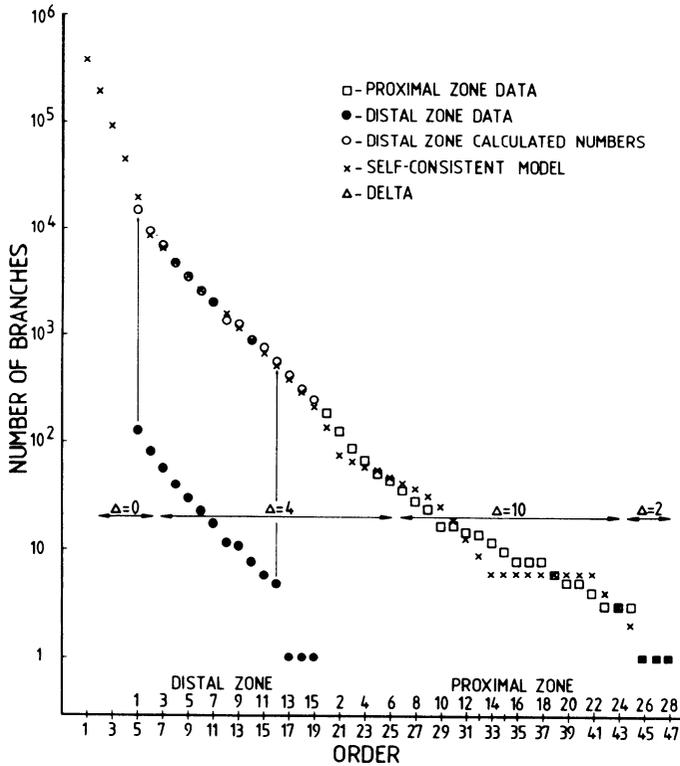


FIG. 3. Numbers of branches in each order in data and model. Number is shown on logarithmic scale on ordinate. For distal zone numbers are total from seven samples. Three scales of order are shown on abscissa, one each for proximal and distal zone data, and one (lower) for model. Range of orders for each value of delta in model is indicated by horizontal arrows. Vertical arrows indicate how distal zone sample data are joined onto proximal zone data by multiplying by a constant factor, thereby moving points to a higher, parallel position such that ends of two plots meet.

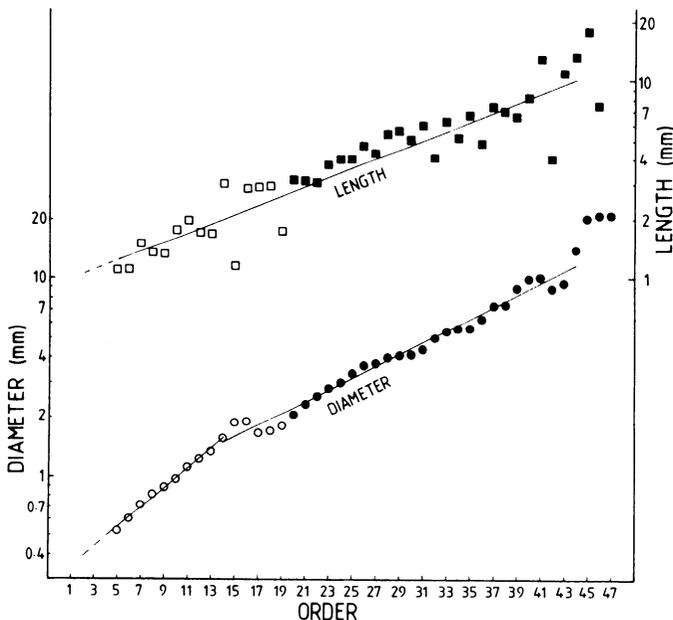


FIG. 4. Mean dimensions of branches in data and model. Diameter and length are shown on separate logarithmic scales on ordinate. Open symbols are distal zone data and closed symbols are proximal zone data. Continuous lines are lines of best fit up to order 44, and these define dimensions used in model. Dashed lines are extrapolations to determine dimensions of respiratory bronchioles, orders 2-4. Data for orders 45-47 are used unchanged in model.

these orders had to be estimated as already described. For orders 1-12 it was assumed that the sample correctly represented the proportion of each order in the total, and thus on a semilog plot the sample data and the total numbers would lie on parallel lines (Fig. 3). The correct position on the vertical scale of this parallel line is found by joining its lower end to the upper end of the data for the higher orders. This is done by multiplying the distal zone data by a constant factor (4, 6) such that the point for order 12 coincides with that already estimated. This factor is (number estimated)/(number in sample) or  $585/5$ . When the other data points are multiplied by the same factor, the resulting points lie on a line parallel to the data and connected with the plot for the higher orders. In other words, the sample is taken to represent  $5/585$  or  $1/117$  of the total distal zone numbers.

Construction of the Model

Before considering the model and its elaboration it is necessary to consider the simple principles of ordering asymmetrical dichotomously branching trees. Figure 1 shows such a tree, in which the peripheral branches are defined as order 1; two of these meet to form an order 2 branch. Subsequently the parent branch is defined as being one order higher than the higher ordered of the two meeting daughter branches. The two daughter branches at any bifurcation may thus be of equal order or of different order, and the degree of asymmetry at the bifurcation may be defined by delta, the difference between the orders of the two daughter branches. When  $\delta = 0$  the branching is symmetrical (with respect to orders) as shown in Fig. 5A. The least degree of asymmetry is when  $\delta = 1$ , as in Fig. 5B, and greater asymmetry is represented by larger values of delta (Fig. 5C). It should be noted that order 2 can only branch with  $\delta = 0$ , so that the branching ratio between any one order 2 branch and its daughters must be exactly 2.0, but in the total tree it may well be more than 2.0 because order 1 branches can arise directly from, for example, order 3 branches.

The concepts used in the development of this model may perhaps best be understood in relation to previous work on bronchial tree models. Probably the best known is Weibel's symmetrical model (14) in which every branch in a given generation bifurcates symmetrically with  $\delta = 0$ . As the number of generations, 23, is known, it is possible to draw a diagram of the branching pattern of the whole tree because the connectivity, the way the

TABLE 2. Branching ratio and delta in each zone

Zone	Order in Zone Data	Order in Model	Diam in Model, cm	$R_b$	Corresponding Value of Delta
Proximal	6-25	25-44	0.314 - 1.15	1.150	14
Proximal	1-6	20-25	0.223 - 0.314	1.330	4
Distal	1-12	5-16	0.0554 - 0.169	1.338	4

$R_b$ , branching ratio. Note that orders 26-28 in the proximal zone (45-47 in the model) and 13-15 in the distal zone (17-19 in the model) were not used in the calculation of  $R_b$  because there was only one example of each in the data.

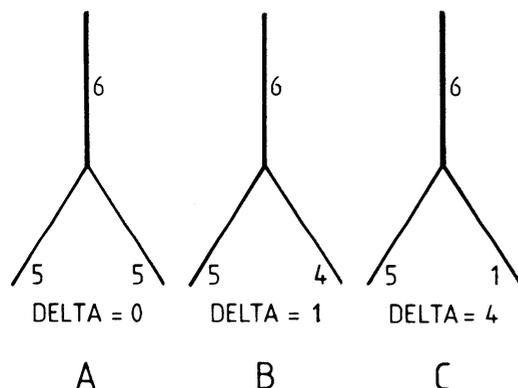


FIG. 5. Different values of delta (difference between orders of two daughter branches) at a dichotomous branching. A: Symmetrical branching. B: Least degree of asymmetrical branching. C: More markedly asymmetrical branching.

branches join together, is defined. Thus for example, a generation 8 branch has a generation 7 parent and two generation 9 daughter branches. Furthermore, because the dimensions of the branches in any given generation are all the same, it is possible to perform calculations, e.g., of volume or resistance, based on just one branch and then apply them to all the other branches of that generation. It is this property, namely that all branches in any given generation are identical, that is the main reason why Weibel's model has found such wide acceptance. Similarly, in our asymmetrical model, all the branches of any one generation or order have identical dimensions and divide with the same single value of delta. In the descriptions of the asymmetrical bronchial tree of the dog by Ross (9) and of humans by Horsfield and Cumming (5) delta was not defined, and so this property is lacking. This is the reason why these data have proved to be so difficult to use for physiological calculations, although both papers contain detailed information pertaining to the asymmetry of the bronchial tree. Horsfield et al. (7) attempted to improve this by describing two asymmetrical models of the human bronchial tree based on the use of delta within lobes, or within bronchopulmonary segments. More recently, Fredberg and Hoenig (1) developed the lobar model into a model of the whole bronchial tree.

While Weibel's symmetrical model is easy to understand, the mode of branching in asymmetrical trees, in which branches have been classified by order defined by delta, requires further explanation. Broadly speaking, there are two main types, one in which the value of delta is constant throughout, and the other in which it differs from order to order. The simplest example of the first type is when  $\delta = 1$  throughout, which gives rise to the tree shown in Fig. 6. Similar trees, with  $\delta = 2$  or more at each bifurcation, can easily be drawn in principle, although in practice there are difficulties in ending the tree at the lower orders (see dashed lines in Fig. 6). An example of the second type, with delta alternating between 0 and 1 in successive orders, is shown in Fig. 7A. It may be seen that the branches of order 3, for example, each give rise to identical patterns of branching, and that therefore an abbreviated diagram can be used to represent the tree as shown in Fig. 7B. Once this concept has been grasped it is easy to see that the list of the values of

delta at each order (inset Fig. 7) conveys the same information: given the list, the whole tree can be drawn without difficulty. This is the way in which the model of the dog's bronchial tree, developed below, will be presented in Table 3.

In making such a model the problem is to represent the range of asymmetry found at a given order in the data by an average asymmetry defined by a single value of delta. Elaboration of the model then falls into two stages: first, the definition from the data of the number of orders, number of branches, and their dimensions; and second, the determination of a pattern of branching that gives numbers of branches similar to the data. In the previously published lung model based on lobes (7) this method gave a distribution of the number of orders down every pathway in the model similar to that found in the data.

*Number of orders.* The 28 orders in the proximal zone data and 15 orders in the distal zone data were joined together to give 43 orders of conducting airways. To these were added three orders of respiratory bronchioles and one order to represent the alveolar duct system,

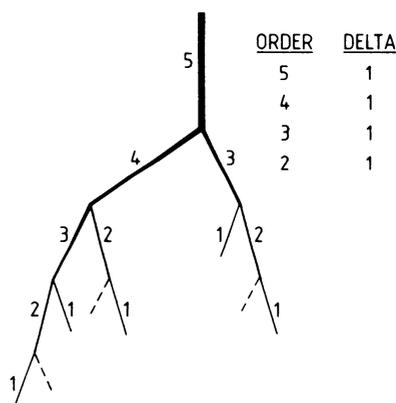


FIG. 6. Asymmetrical branching tree defined by  $\delta = 1$  throughout. Branches are numbered by order. There is some difficulty in bringing this pattern to termination, as indicated by dashed lines.

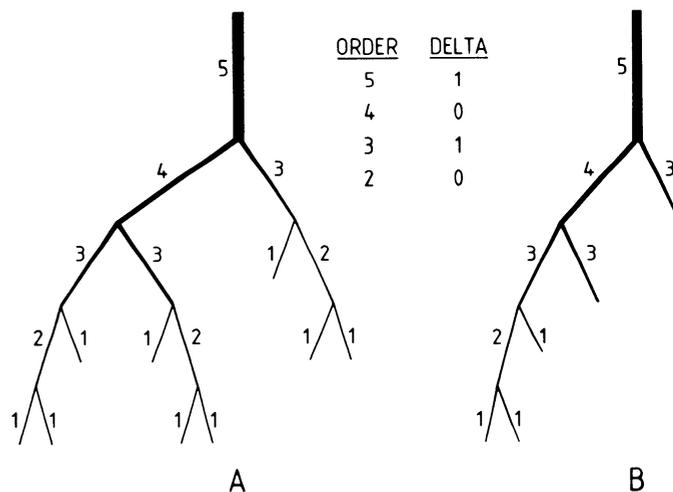


FIG. 7. A: asymmetrical branching tree numbered by order. Branching pattern is defined by values of delta alternating between 1 and 0. B: because patterns of branching subtended by all branches of any one order are identical, tree can be drawn in abbreviated form in which only one example of a bifurcation of each order is shown. Same information is conveyed by list of values of delta at each order.

giving a total of 47 orders. Because of the lack of data in the literature on detailed dimensions of dog alveolar ducts and the highly irregular branching pattern they have (11), it was considered impossible to model them adequately at the present time. They were therefore left as one order, which can be utilized or not as required by anyone using the model. For many purposes a lumped parameter might suffice to represent the alveolar ducts.

**Dimensions.** The length and diameter data are shown plotted against order in Fig. 4. Lengths in the model were determined from the regression equation of the line of best fit for orders 5–44. Diameters in the model were determined from the regression equations of two lines of best fit, one for orders 5–14, and one for orders 14–44. Diameters of orders 45–47 and lengths of orders 45 and 46 were taken directly from the data. The length of the trachea (order 47) was taken to be 20 cm. Dimensions of the respiratory bronchioles were determined by extrapolation (Fig. 4). A check was made on the peripheral samples to confirm that the values thus obtained were similar to those for the respiratory bronchioles on the cast.

**Pattern of branching.** In a branching tree with a defined constant value of delta, the branching ratio may be calculated (4, 7). Thus if  $\delta = 0$ ,  $R_b = 2.0$ ; if  $\delta = 1$ ,  $R_b = 1.618$ ; if  $\delta = 3$ ,  $R_b = 1.466$ , and so on. Conversely, if  $R_b$  is known a value of delta can be found that most nearly gives that branching ratio. For the model, values of  $R_b$  of 1.33 and 1.338 are best represented by  $\delta = 4$ , which is therefore used for orders 7–25. For orders 26–44  $R_b = 1.15$ , which is best represented by  $\delta = 14$ . However, this value proved to be unsatisfactory in

practice because too many orders (about 23) are required for the theoretical value of  $R_b$  to become established. It was found by trial and error that a value of 10 gave the best fit to the data as judged by eye, so this was used instead. At the top end of the model delta is inconsistent in the data, but an average value of 2 was similarly found to give the best fit. Finally, at the distal end, branching of respiratory bronchioles is more nearly asymmetrical. From the data the number of terminal bronchioles was estimated to be 15,327, compared with the estimate of 20,600 by Ross (9), and 25,736 from a Strahler ordered model of the dog lung (6). The present model was designed to have about 20,000 terminal bronchioles (order 5), an average value from these three estimates, and approximately eight times as many third-order respiratory bronchioles (order 2). By attributing  $\delta = 0$  for orders 2–6 these criteria were met reasonably well. The numbers obtained are plotted in Fig. 3 and the entire model is given in Table 3.

## DISCUSSION

The model that has been developed is designed to represent the average asymmetry of the real bronchial tree. Judging by the similarities of the numbers of branches in the data and the model (Fig. 3) it probably does this reasonably well. Orders 32–34 show the least good correspondence, due to the rather artificial way in which the branching tree develops with order when delta is high. Because every pathway in the model is defined, it would be possible to calculate the distribution of path lengths, though this has not been attempted. The shortest path from carina to respiratory zone is 4 cm and the longest 20 cm, compared with the data of Ross (9) in which the range is from 2–14 cm. Probably he used smaller lungs because the diameters he gives are also smaller. Ross expressed the asymmetry of the lung in terms of path length, but it is difficult to tell from his paper how many orders of branching were found, so that direct comparison is difficult.

This model emphasizes one of the important differences between Horsfield (6, 7) and Strahler (12, 13) ordering systems, i.e., that the number of Strahler orders is always less. A dog lung was found to have 10 Strahler orders of conducting airways (6) compared with 43 Horsfield orders in the data given here. The lesser degree of asymmetry found in human lungs compared with the dog lungs is also demonstrated by the human lung having only 28 orders of conducting airways (7) but a similar number of terminal bronchioles (between  $2 \times 10^4$  and  $3 \times 10^4$ ).

The data from only one pair of lungs, obtained from a dog weighing 25 kg, were used in constructing this model. Probably the best way to scale dimensions for dogs of different weights is in proportion to cube root of body weight, as has been shown in dogs at different stages of growth (3). However, in a series of adult dog lung casts (15) dimension/weight<sup>1/3</sup> showed considerable variation for the larger airways, although less than dimension alone.

The value of the model lies in the fact that not only are all branches of a given order identical, but the seg-

TABLE 3. *Asymmetrical model of airways of the dog based on delta*

Order	Delta	Diam. cm	Length, cm	Order	Delta	Diam. cm	Length, cm
47	2	2.10	20.00	23	4	0.274	0.328
46	2	2.10	0.75	22	4	0.255	0.311
45	2	2.03	1.80	21	4	0.238	0.294
44	10	1.15	1.01	20	4	0.223	0.279
43	10	1.08	0.961	19	4	0.208	0.264
42	10	1.01	0.911	18	4	0.194	0.251
41	10	0.939	0.863	17	4	0.181	0.237
40	10	0.876	0.818	16	4	0.169	0.225
39	10	0.818	0.775	15	4	0.158	0.213
38	10	0.764	0.734	14	4	0.148	0.202
37	10	0.714	0.696	13	4	0.139	0.191
36	10	0.666	0.660	12	4	0.124	0.181
35	10	0.622	0.625	11	4	0.110	0.172
34	10	0.581	0.592	10	4	0.0983	0.163
33	10	0.543	0.561	9	4	0.0876	0.154
32	10	0.507	0.532	8	4	0.0781	0.146
31	10	0.473	0.504	7	4	0.0697	0.139
30	10	0.442	0.478	6	0	0.0621	0.131
29	10	0.413	0.453	5	0	0.0554	0.125
28	10	0.385	0.429	4	0	0.0494	0.118
27	10	0.360	0.407	3	0	0.0440	0.112
26	10	0.336	0.385	2	0	0.0393	0.106
25	4	0.314	0.365	1			
24	4	0.293	0.346				

Order 47, trachea; order 5, terminal bronchioles; orders 2–4, respiratory bronchioles; order 1, alveolar ducts system. Delta is the difference in order between two daughter branches. The branching pattern of the model can be reconstructed from the values of delta, as shown in Fig. 7.

ments of tree that they subtend are also identical. This property greatly facilitates calculation of physiological variables such as, for example, input impedance (1). For any branch the input impedance is a function of the input impedances of its daughter branches. By starting at the lowest orders and working up, storing the calculated value for each order in turn, when a branch of order  $n$  is reached reference may easily be made to the values

already calculated for its daughter branches, namely orders  $n - 1$  and  $n - (1 + \delta)$ . Thus the calculation need be done only once for each order. By comparison, if the same calculations were done on the raw data of Raabe et al. (8) or Horsfield and Cumming (5), they would have to be done separately for every branch.

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